1. (10 points). (a) Find the MM estimator of λ in the Poisson distribution and prove that it is unbiased given a random sample $X_i \overset{\text{iid}}{\sim} \mathcal{P}(\lambda)$, $i = 1, 2, ..., n$. (b) Prove that $\sum_{i=1}^n (X_i - \overline{X})^2/(n-1)$, as another estimator of λ, is unbiased. (c) Use simulations to compare the performance of both estimators for a sequence of λ at fixed n using the empirical MSE computed as the average of $(\widehat{\lambda}_k - \lambda)^2$ where k is the kth simulation. [Hint: Use n as an argument of your function and plot the two MSEs on the same graph over sequence of λ from 0.5 to 3 with the step 0.5; show the graph with $n = 5$ and the number of simualtions 10,000. Make necessary notations.]

2. (5 points). Let in the simple linear regression $x_k = \min x_i$ and $x_l = \max x_i$. Define an estimator of the slope as $\widetilde{\beta} = (Y_l - Y_k)/(x_l - x_k)$. (a) Prove that this estimator is unbiased and provide a geometric illustration. (b) Prove that $var(\widehat{\beta}) \leq var(\widetilde{\beta})$.

3. (15 points). Modify function **simlin** to demonstrate by simulations that (1) $var(\widehat{\beta}) = \sigma^2/\sum_{i=1}^n (x_i - \overline{x})^2$, (2) $\sqrt{\sum_{i=1}^n (x_i - \overline{x})^2}(\widehat{\beta} - \beta)/\widehat{\sigma}$ has t-distribution with $n-2$ df, and (3) $\widehat{\beta} \pm t_{1-\alpha/2}$SE covers the true β with probability $1 - \alpha$. [Hint: Print out the thoretical and simulation-derived variance of the slope; to demonstrate the t-distribution superimpose the empirical cdf from simulations with the theoretical t-distribution, use $\lambda = 0.95$, `nSim=10000`, and `summary(outLM)$sigma` to extract $\widehat{\sigma}$.]