Prove that $E(F(X)) = 1/2$ where F is the cdf of X. Use simulations with the normal distribution to confirm.

Solution. We use the fact that $F(X)$ has a unifirm distribution on $(0, 1)$. This implies $E(F(X)) = 1/2$. The following R code is used to confirm:

```
X=rnorm(100000)
e.emp=mean(pnorm(X))
cat("Simulation-derived F(FI(X)) =",e.emp)
```