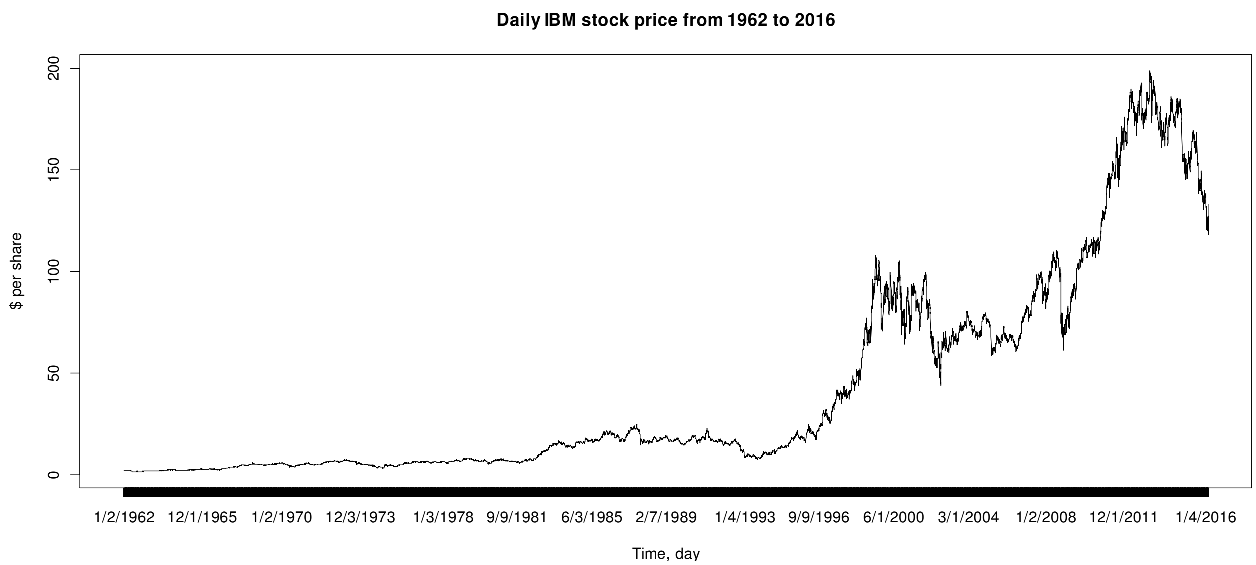
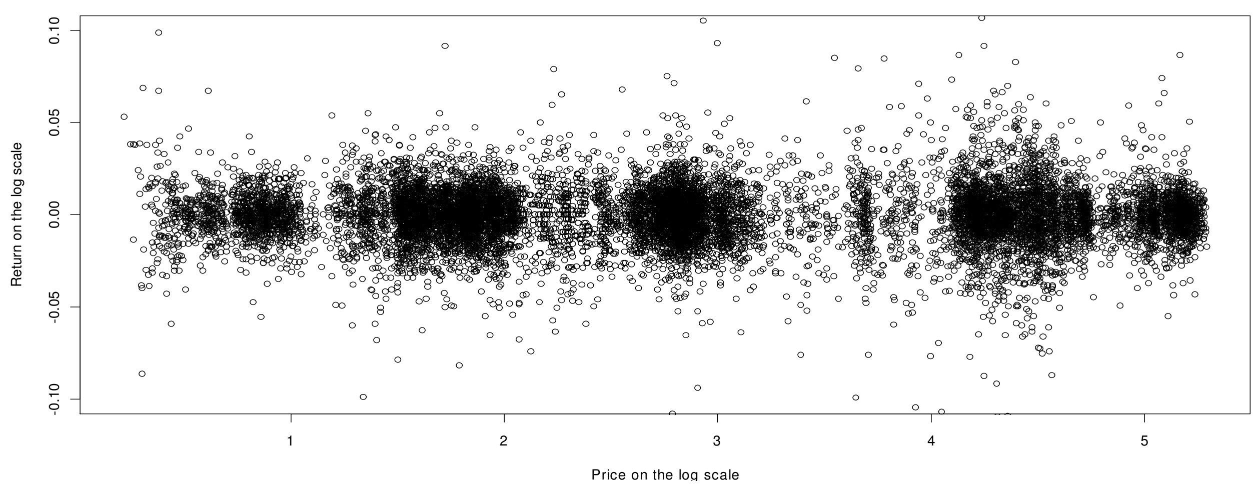
Homework 6

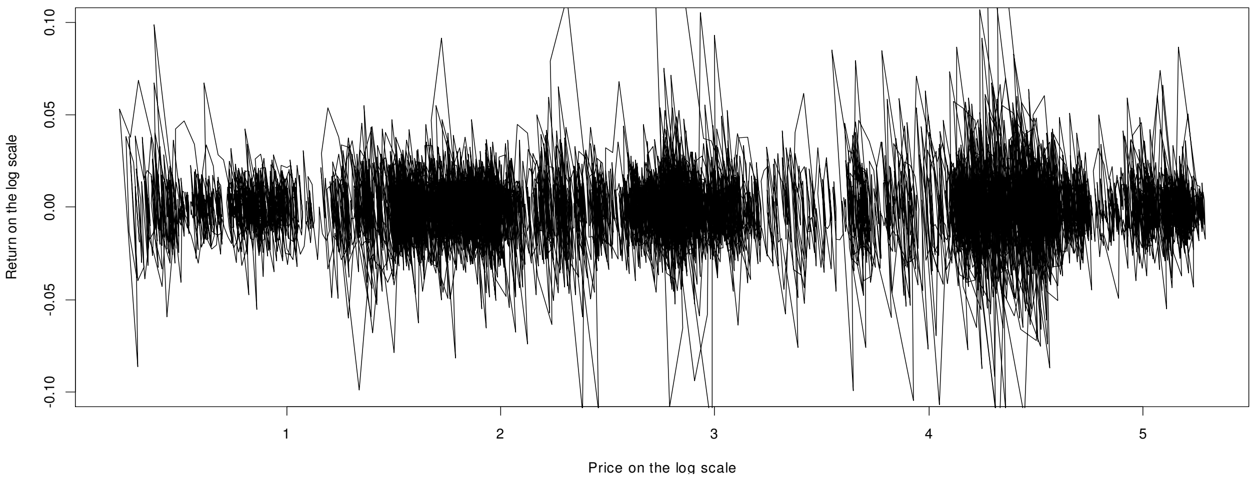
Problem 5.6.4.

(1)

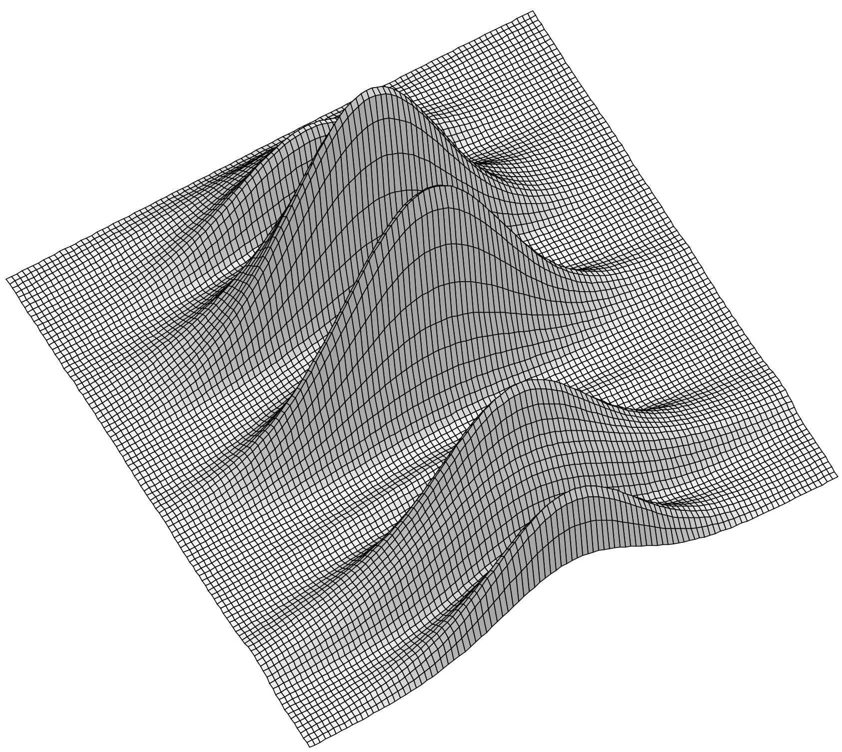


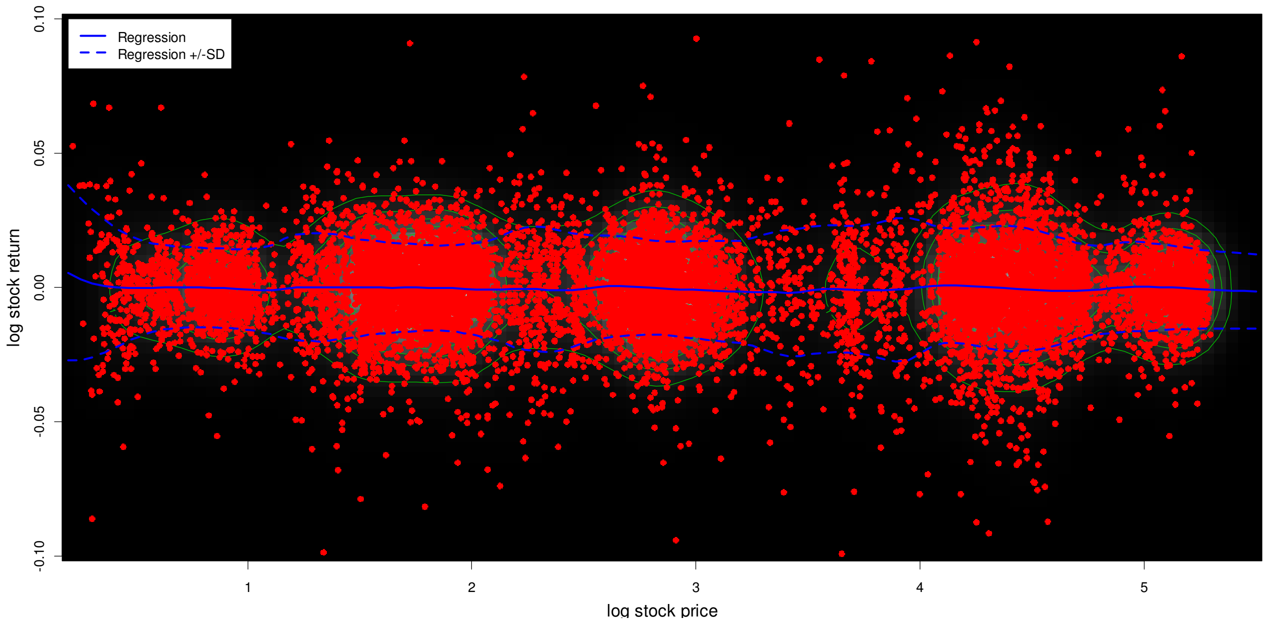
(2)




Return on the log scale
0.10
0.05
0.00
-0.05
-0.10
1
2
3
4
5
Price on the log scale

(3)





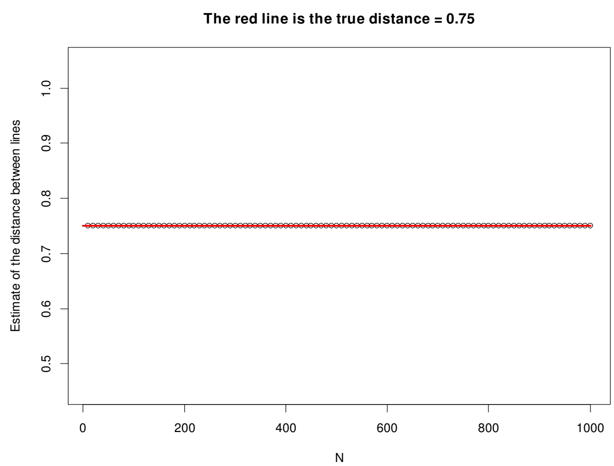

```
ibm2d=function(job=1,hx=.1,hy=.01,N=100)
{
 dump("ibm2d","c:\\M4017\\ibm2d.r")
 d=read.csv("c:\\M4017\\IBM_daily.csv")
 sp=d[,7];n=length(sp);n1=n-1
 y=log(sp[2:n]/sp[1:n1])
 x=log(sp[1:n1])
 i=1:n1
 x=x[seq(from=n1,to=1,by=-1)];y=y[seq(from=n1,to=1,by=-1)]
 sp=sp[seq(from=n,to=1,by=-1)]
 da=d[,1];da=da[seq(from=n,to=1,by=-1)]
 if(job==1)
 {
 par(mfrow=c(1,1),mar=c(4,4,4,1))
 plot(1:n,sp,type="l",xaxt="n",xlab="Time, day",ylab="$ per share",
    main="Daily IBM stock price from 1962 to 2016")
 axis(side=1,at=1:1:n,labels=da)
 }
if(job==2)
{
 plot(x,y,type="p",ylim=c(-.1,.1),xlab="Price on the log scale",ylab="Return
on the log scale")
}
if(job==2.1)
{
 plot(x,y,type="l",ylim=c(-.1,.1),xlab="Price on the log scale",ylab="Return
on the log scale")
}
```

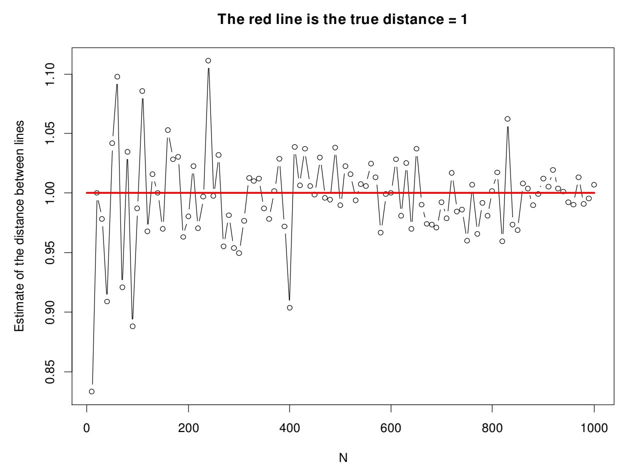
```
if(job==3)
{
 par(mfrow=c(1,1),mar=c(0,0,0,0))
 x.grid=seq(from=min(x),max(x),length=N);y.grid=seq(from=-.05,to=.05,length=N)
 fxyOR=bvn.density.my(x.data=x,y.data=y,x=x.grid,y=y.grid,hx=hx,hy=hy)
 kd2=persp(x.grid,y.grid,fxyOR,theta=60,phi=70,r=100,box=F,
    col=gray(.98),shade=.7,ltheta = -15, lphi = 100)
}
if(job==4)
{
 par(mfrow=c(1,1),mar=c(3.5,3.5,1,1))
 hx=.1;hy=.01;Nx=100;Ny=50
 yx=cbind(y,x)
 n=nrow(yx)
 x=seq(from=.2,to=5.5,length=Nx)
 y=seq(from=-.1,to=.1,length=Ny)
 fxy=bvn.density.my(x.data=yx[,2],y.data=yx[,1],x=x,y=y,hx=hx,hy=hy)
 image(x,y,fxy,col=gray((0:255)/255),xlab="",ylab="")
 mtext(side=1,"log stock price",cex=1.25,line=2.5)
 mtext(side=2,"log stock return",cex=1.25,line=2.5)
 contour(x,y,fxy,add=T,col=3)
 points(yx[,2],yx[,1],col=2,pch=16)
 ro=round(cor(yx[,1],yx[,2]),2)
 text(4,-6,paste("ro=",ro,sep=""),col="white",cex=1.25)

 eNx=rep(1,Nx);en=rep(1,n)
 M=(x%*%t(en)-eNx%*%t(yx[,2]))/hx
 fi=dnorm(M)
 den=fi%*%en
 Eyx=((eNx%*%t(yx[,1])+ro*hy*M)*fi)%*%en/den
 lines(x,Eyx,col=4,lwd=3)

 va=(1-ro^2)*hy^2+((Eyx%*%t(en)-eNx%*%t(yx[,1])-ro*hy*M)^2*fi)%*%en/den
 lines(x,Eyx+sqrt(va),col=4,lwd=3,lty=2)
 lines(x,Eyx-sqrt(va),col=4,lwd=3,lty=2)
 legend(.2,.1,c("Regression","Regression +/-SD"),col=4,lty=1:2,lwd=3,bg="white")
}


}
```

6.1.1. Whit the default distance between lines equal the diameter of the penny (0.75) penny always intersects a line.

```
 penny=function(d=.75)
{
```

```
# d=distance between lines
dump("penny","c:\\M4017\\penny.r")
N=seq(from=10,to=1000,by=10)
K=length(N)
d.est=rep(NA,K)
for(i in 1:K)
{
 X=runif(N[i],min=0,max=d)
 d.est[i]=.75/mean(X-.75/2<0 | X+.75/2>d)
}
plot(N,d.est,type="b",xlab="N",ylab="Estimate of the distance between lines")
title(paste("The red line is the true distance =",d))
segments(-1,d,1000,d,col=2,lwd=3)
}
```





6.2.4. We have $E(X) = (\tau + \theta)/2$ and $var(X) = (\theta - \tau)^2/12$. Find estimators of θ and τ by equating the empirical and theoretical moments.

$$\tau + \theta = 2\overline{X}, \quad \theta - \tau = \sqrt{12}\widehat{\sigma},$$

where $\widehat{\sigma} = \sqrt{n^{-1}\sum_{i=1}^{n}(X_i - \overline{X})^2}$, that yields the MM estimators

$$\widehat{\theta}_{MM} = \overline{X} + \sqrt{3}\widehat{\sigma}, \quad \widehat{\tau}_{MM} = \overline{X} - \sqrt{3}\widehat{\sigma}.$$

6.3.7. (a) It is not true. First of all we note that $\widehat{\theta}_3 = (\widehat{\theta}_1 + \widehat{\theta}_2)/2$ is unbiased because $E(\widehat{\theta}_3) = (E(\widehat{\theta}_1) + E(\widehat{\theta}_2))/2 = (\theta + \theta)/2 = \theta$. This means that instead of MSE we can operate with var. If ρ is the correlation coefficient then $var(\widehat{\theta}_3) = (\sigma_1^2 + \sigma_2^2)/4 + \rho\sigma_1\sigma_2/2$. Consider the case when $\sigma_1 = 1$ and $\sigma_2 = 2$. Then $var(\widehat{\theta}_3) = 5/4 + \rho > 1 = \sigma_1^2 = var(\theta_1)$ if $\rho > 0$. That is, we found $\widehat{\theta}_1$ and $\widehat{\theta}_2$ such that $var(\widehat{\theta}_3) > var(\widehat{\theta}_1)$. (b) We have

$$var(\widehat{\theta}_3) = \lambda^2\sigma_1^2 + 2\lambda(1 - \lambda)\rho\sigma_1\sigma_2 + (1 - \lambda)^2\sigma_2^2.$$

Differerntiating with respect to λ and equating to zero we obtain the optimal

$$\lambda = \frac{\sigma_2^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}.$$

This optimal λ will not reduce MSE if λ does not exist, i.e. when the denominator is zero, $\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2 = 0$ which happens only if $\rho = 1$ because

$$\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2 \geq \sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2 = (\sigma_1 - \sigma_2)^2 \geq 0$$

and the equality happens iff $\rho = 1$ and $\sigma_1 = \sigma_2$. The answer: the optimal combination does not reduce MSE when the two estimators are actually the same.