

# Math 40 Probability and Statistical Inference

## Homework 01

Winter 2021

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Due Jan 19, 2021 11:59 pm (EDT)

Do the following exercises of the textbook. Show all steps to get your answers. Specify the problems you discussed with other students (including names).

5 points for each problem.

- 1.3.2 (in this problem, you do not need to explain your answer) Express  $p_i$  (the probability of having  $X = x_i$ ) in terms of cdf (cumulative distribution function)
- 1.4.1 Introduce a function  $M(a) = E[(X - a)^2]$ . (a) Express this function through var and  $\mu$  (the mean). (b) Prove that this function takes the minimal value at  $a = \mu$ . (c) Find the minimum of  $M$ .
- 1.6.1 Derive the formula  $\sum_{m=0}^n \binom{n}{m} = 2^n$  from the binomial probability

$$Pr(X = m) = \binom{n}{m} p^m (1 - p)^{n-m}.$$

- 1.6.6 Sixteen players of two genders sign up for a tournament. What is the probability that there will be eight men and women? Provide a theoretical answer and confirm via simulations (simulation is optional).
- 1.6.8 The chance that a person will develop lung cancer in his/her lifetime is about 1 in 15. What is the probability that in a small village with 100 people, (a) there are no individuals with lung cancer, and (b) there is only one case? (Optional) in both cases check your answer with simulations.
- 1.7.3 Assume that the number of children in the family follows a Poisson distribution with  $\lambda = 2.4$ . Find the minimum number of toys that ensures each child gets a toy with probability 99%.
- 1.7.5 Prove that for the Poisson distribution,  $var(X) = \lambda$  using the formula  $\sigma^2 = E(X^2) - \mu^2$ .
- 1.7.9 10% of families have no children. Assuming that the number of children in the family follows a Poisson distribution, estimate the average number of children in the family.

9. 1.7.10 What is the probability that a randomly chosen family with the number of children two or more has children of the same gender? [Hint: Assume a Poisson distribution and use conditional probability].
10. 1.7.13 The number of children in the family follows a Poisson distribution with  $\lambda = 1.8$ . Six families with kids are invited to a birthday party. What is the probability that more than 15 kids come to the party. Give the answer under two scenarios: (a) family brings all kids they have, and (b) the probability that they bring a child is 0.8. (optional) Write a simulation program to check your answer.