Math 40 Probability and Statistical Inference
Homework 02

Winter 2021

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Due Jan 26, 2021 11:59 pm (EDT)

Do the following exercises of the textbook. Show all steps to get your answers. Specify the problems you discussed with other students (including names).

5 points for each problem.

1. 2.1.3 Let X be a continuous random variable and F be its strictly increasing cdf. Prove that random variable $F(X)$ and $1-F(X)$ have the same distribution. Hint: Prove that the cdf of $F(X)$ is x.
2. 2.1.10 The density of a random variable is defined as $c\sin(x)$ for $0 < x < \pi$ and 0 elsewhere. Find c and the cdf.
3. 2.2.14 Prove that $E(F(X)) = 1/2$ where F is the cdf of X. Hint: assume continuous distribution and apply change of variable.
4. 2.2.15 Prove that the minimum of $\int_{-\infty}^{a} F(x)dx + \int_{a}^{\infty}(1-F(x))dx$ attains when a is the median.
5. 2.3.7 Find the kurtosis for $\mathcal{R}(a,b)$
6. 2.3.8 (modified) Two friends come to the bus stop at random times between 1 and 2 pm. The bus arrives every 15 minutes. Find the probability that the friends end up on the same bus (do not use simulations). Hint: think about discrete random variables with 4 outcomes.
7. 2.4.7 In the telephone example (example 2.16), what is a better probability to talk with Bill: (a) wait for the first 5 minutes or (b) wait for the call from 10:10 to 10:40?
8. 2.4.10 (Refer example 2.19) Modified: Assuming that we just had an earthquake, find the probability of having another earthquake within a week. Calculate the probabilities for the three regions.
9. 2.5.2 (modified) Find the MGF of Bernoulli(p). Use this result to find the MGF of the binomial distribution with (n,p). Hint: the MGF of the sum of independent random variables is the product of individual MGFs.
10. 2.5.7 Prove that if $M(t)$ is the MGF of a random variable X, then the MGF of $a+bX$ is $e^{at}M(bt)$.