Math 40 Probability and Statistical Inference Homework 03

Winter 2021

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Due Feb 1, 2021 11:59 pm (EDT)

Do the following exercises of the textbook. Show all steps to get your answers. Specify the problems you discussed with other students (including names).

5 points for each problem.

1. 2.6.2 The quality-control department investigates the reliability of a product whose time to failure follows a gamma distribution. It was observed that the mean was six years with the standard deviation of three years. What is the probability that after 10 years the product will be still functioning?

2. 2.6.9 There are five friends. The first friend heard a rumor and he/she called the second friend and that friend called the third friend, and so on. Assume that each friend calls once and the average time to call is one hour. What is the most probable time when the last friend hears the rumor?

3. 2.7.6 Prove that $Pr(|X-a|<\tau)$ takes maximum when $a=\mu$ where $X\sim\mathcal{N}(\mu,\sigma^2)$

4. 2.7.10 In Example 2.34, find and compute the probability that the volume of the ball is within ± 0.1 of the nominal volume.

5. 2.8.5 For an exponential distribution with λ, compute (a) $Pr(Z>t)$ and (b) $\frac{E(Z)}{t}$ for $t>0$.

6. LLN The following is a statement from a student who took Math 40. Discuss which step ((i) (iii)) is (are) incorrect.

   "Let $X1, X_2, ..., X_n$ be IID. From this sample, we want to estimate the variance of X_i, σ^2_{true} which is unknown. My estimation for the variance is the following

   $$\hat{\sigma}^2 = \frac{1}{n}\sum_i^n (X-\overline{X})^2, \quad \textit{where} \quad \overline{X} = \frac{1}{n}\sum_i^n X_i.$$

   This estimator is unbiased, that is, the expected value of $\hat{\sigma}^2$ is the true variance σ^2_{true}. My reasoning is that

   i) as $n\to\infty$, $\overline{X}\to\mu_{true}$ from LLN.

   ii) Thus, $E((X-\overline{X})^2)=\sigma^2_{true}$.

   iii) Using LLN again, $\hat{\sigma}^2 \to \frac{n\sigma^2_{trye}}{n} = \sigma^2_{true}$ as $n\to\infty$.

"

7. 2.10.11 Random walk on the line: a drunk walker takes step left or right with probability 1/2. Assuming that one step if 3 ft and he takes a step every second, what is the probability that he will be at a distance 100 ft or more from the place he started after one hour? Use CLT to assess the probability.

8. 2.10.12 Random walk on the plane: a drunk walker takes a step left, right, up, or down with probability 1/4. Assuming that one step is 3ft and he takes a step every second, what is the probability that he will stay within the square $+ \pm 100ft$ centered on the place he started after one hour? Use CLT to assess the probability.

9. 2.12.4 Find the expected value of the random variable with the density specified by (2.70) (in example 2.63)

10. 2.12.12 The radius of the sphere is measure with $1\% error$. What percentage measurement error does this imply for the volume?