Math 46: Applied Math: Homework 7

due Wed May 20 . . . but best if do relevant questions after each lecture

In #7b,c you'll need the Fourier sine series on $[0, \pi]$,

$$x(\pi - x) = \frac{8}{\pi}\left[\sin x + \frac{\sin 3x}{3^3} + \frac{\sin 5x}{5^3} + \cdots\right]$$

How did I quickly work out the coefficients? Via these Maple commands—try it!

```
assume(n,integer);
assume(n,odd);                                  # by symmetry you know only odd sines contribute
int(sin(n*x)*x*(Pi-x), x=0..Pi) / int(sin(n*x)**2, x=0..Pi);      # coeffs (f,f_n)/||f_n||^2
```

p.243-247: #4. c. Keep taking derivatives and cancelling stuff until you've transformed to a (very familiar) SLP.

d.

a. Apply the addition formula, then start guessing eigenfunctions.

#7. [ask if you didn't get the eigenvalues and eigenfunctions from #4 c. Each time test if μ is an eigenvalue before proceeding]. This is a nice question: each part gives a different scenario in terms of solvability. For part d please use a right-hand side of $\sin 2x$ instead of $\cos 2x$.

Congratulations on solving your first symmetric Fredholm integral equations, ∞-dimensional problems!

A) The 1D periodic image function f on $[-\pi, \pi]$ is blurred by the periodic kernel defined on $[-\pi, \pi]$ by an 'aperture function' $k(s) = 1$ for $|s| < \pi/2$, zero otherwise. i) Give a formula for what blurring does to the Fourier coefficients $a_0, a_1, \ldots, b_1, \ldots$ of f. ii) Give a formula for the deblurred image function in terms of the Fourier coefficients $A_0, A_1, \ldots, B_1, \ldots$ of a measured blurry image. [reconstruct only the possible coefficients]. iii) Measurement brings an error of 0.01 into all Fourier coefficients of the blurry image. How many coefficients can be reconstructed if the error of any reconstructed coefficient should not exceed 0.3?

B) Demonstrate that $v(x)$ defined on p.250 indeed solves $Lv = f$, thus the theorem giving the formula for Green's function is correct. Sorry about the algebra, but I care you see how W is cancelled from the denominator to leave f. BONUS: demonstrate one BC is satisfied.

p.257-258: #1. [view the LHS as a differential operator]

#2. You should get an explicit expression for $u(x)$ in terms of f, and state for what f a solution exists. [Hint: use an expansion in eigenfunctions of L, or Thm 4.23]

#5. Appreciate the power of what you've just done: a closed-form expression for the solution to arbitrary heat source function in arbitrary conductivity medium!

#7. Sniff around for similar-looking expressions.

#8. You should get an integral operator. Please write *two* different forms for its kernel: one will involve a sum and the other won't. [Hint: for the sum version use p.224-226 #7 and it's easy]