MATH 46 WORKSHEET : Asymptotic analysis

Barnett
4/10/09

A) is $\tan \varepsilon = o(\varepsilon)$ as $\varepsilon \to 0$?

is $\tan \varepsilon = O(\varepsilon)$ as $\varepsilon \to 0$? [Hint use graph first, prove later]

B) Is $f(t,\varepsilon) := \varepsilon \tan t$ uniformly convergent to zero on $(0, \pi/4)$ as $\varepsilon \to 0$?
[Hint: graph it vs t].

on $(0, \pi/2)$?

does $f(t,\varepsilon)$ converge pointwise to zero on $(0, \pi/2)$?

C) Rearrange the terms to form a correct asymptotic sequence as $\varepsilon \to 0$:

$$y(t) = \varepsilon^{1/2} y_0(t) + \frac{1}{\varepsilon} y_1(t) + \ln\varepsilon \, y_2(t) + y_3(t) + \varepsilon^2 \ln\varepsilon \, y_4(t) + \varepsilon^2 y_5(t) + \varepsilon^2 \ln^2\varepsilon \, y_6(t) + \cdots$$

$y_0 \cdots y_6$ are some funcs of t.
You may just use symbols 0, 1, ⋯ 6 :

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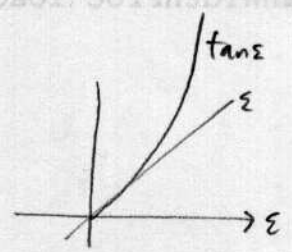
— SOLNS —



A) is $\tan \varepsilon = o(\varepsilon)$ as $\varepsilon \to 0$?

ie is $\frac{f(\varepsilon)}{g(\varepsilon)} = \frac{\tan\varepsilon}{\varepsilon} \xrightarrow[\text{l'Hôpital}]{\lim_{\varepsilon\to 0}} \frac{\sec^2\varepsilon}{1}\Big|_{\varepsilon=0} = \frac{1}{1} = 1$ so, no, not 'little o'.

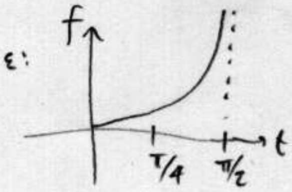
is $\tan\varepsilon = O(\varepsilon)$ as $\varepsilon \to 0$? [Hint use graph first, prove later]

Yes, by above, since limit of ratio is 1, one may choose an M (>1) s.t. $\forall \varepsilon \in [0, c]$, $\frac{\tan\varepsilon}{\varepsilon} < M$.

B) Is $f(t,\varepsilon) := \varepsilon \tan t$ <u>uniformly</u> convergent to zero on $(0, \pi/4)$ as $\varepsilon \to 0$?

[Hint: graph it vs t].

for fixed ε:



yes

on $(0, \pi/2)$? no

since can't 'squeeze' f in interval $(0, \pi/2)$

does $f(t,\varepsilon)$ converge <u>pointwise</u> to zero on $(0, \pi/2)$?

yes.

for fixed $t \in (0, \pi/2)$, $\tan t$ is a number so $\lim_{\varepsilon\to 0} f(t,\varepsilon) = 0$.

C) Rearrange the terms to form a correct <u>asymptotic sequence</u> as $\varepsilon \to 0$:

$$y(t) = \varepsilon^{1/2} y_0(t) + \frac{1}{\varepsilon} y_1(t) + \ln\varepsilon\, y_2(t) + y_3(t) + \varepsilon^2 \ln\varepsilon\, y_4(t)$$

$$+ \varepsilon^2 y_5(t) + \varepsilon^2 \ln^2\varepsilon\, y_6(t) + \cdots$$

$y_0 \cdots y_6$ are some funcs of t.
You may just use symbols 0, 1, ⋯ 6 :

Ans: $\frac{1}{\varepsilon}$ $\quad \ln\varepsilon$ $\quad 1$ $\quad \varepsilon^{1/2}$ $\quad \varepsilon^2/\ln^2\varepsilon$ $\quad \varepsilon^2/\ln\varepsilon$ $\quad \varepsilon^2$

'biggest as ε→0' ... 'smallest as ε→0'