Math 46: X hour of 5/14/09: Degenerate Fredholm Equations

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We used Section 4.3.3, particularly Thms 4.12 and 4.13, to determine if the following had a solution, and then solve them. We made use of (4.31) a lot to get $u(x)$ once the $\mathbf{c}$ vector was found.

Let K operator have kernel $k(x,y) = \sin x \sin y$, on the interval $[0, \pi]$

First: Find the eigenvalues and eigenfunctions of K:

Use this to solve the following:

1. $Ku - u = \sin 2x$
2. $Ku - u = x$
   (We can use Maple to get the integrals $\int_0^\pi x \sin(nx) dx = \pi(-1)^{n+1}/n$)
3. $Ku = 3 \sin 2x$
4. $Ku = 3 \sin x$.

Answer key:

A is 1-by-1 matrix with entry $\pi/2$. Spectrum of K is then $\pi/2$ (multiplicity 1, eigenfunction $\sin x$), and 0 (infinite multiplicity, eigenspace all functions orthog to $\{\beta_j\}$, i.e. orthog to $\sin x$)

1. $c_1 = 0$ so $u = -\sin 2x$
2. $c_1 = \frac{\pi}{1-\pi/2}$ so $u = \frac{\pi}{1-\pi/2} \sin x \ - \ x$
3. no solution
4. $u = \frac{6}{\pi} \sin x \ +$ (any function orthogonal to $\sin x$). Infinitely-nonunique solution.