Worksheet #10: WKB eigenvalues

Consider the boundary value problem

$$-y'' = \lambda q(x) y \qquad \text{where } y(0) = y(1) = 0.$$

(1) Transform the equation into the form

$$\epsilon^2 y'' + (k(x))^2 y = 0.$$

What are ϵ and $k(x)$?

(2) Will WKB apply for small or large λ?

(3) Use WKB to give an approximation of the n^{th} eigenvalue λ_n for the problem

$$-\frac{1}{(2-x^2)^2} y'' = \lambda y \qquad \text{where } y(0) = y(1) = 0.$$

(4) What are the WKB eigenfunctions?