Worksheet #13: Volterra integral equations

(1) Convert the following integral equation into an IVP for $u(t)$.

$$\int_0^t yu(y)dy - \alpha u(t) = f(t) \qquad \text{on } 0 \le t \le 1$$

(2) Prove the lemma:

$$\int_a^x \int_a^s f(y)dyds = \int_a^x f(y)(x-y)dy$$

[Hint: Let $F(s) = \int_a^s f(y)dy$ and use integration by parts.]

(3) Convert the IVP

$$\begin{cases} u''(t) + q(t)u(t) &= g(t) \\ u(0) &= A \\ u'(0) &= B \end{cases}$$

into a Volterra integral equation of the form $Ku - \lambda u = f$ where Ku is an integral operator.

(4) Now try to convert

$$u''(t) + p(t)u'(t) + q(t)u(t) = g(t)$$

into a second kind Volterra integral equation.