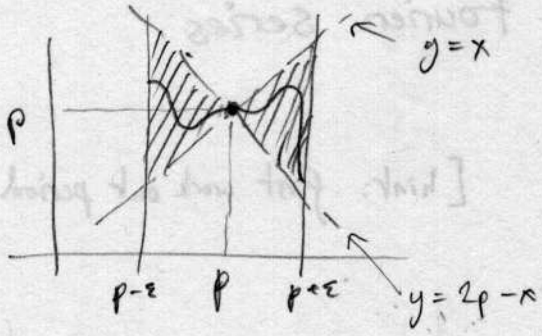
CHAOS MATH 53 HW1 — selected answers Barn 10/3

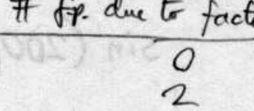
T1.4



graph of f lies in shaded region
$\Rightarrow$ moves closer to p upon iteration
$\Rightarrow$ $N_\varepsilon(p)$ is in the basin of the sink p.

T1.8 see other sheet

T1.11 $f(x) = 3x \pmod 1$ has <u>two</u> fixed pts in $[0,1)$, namely 0 & $1/2$.
$f^k(x) = 3(3\cdots 3x \pmod 1))\pmod 1)\cdots \pmod 1)) = 3^k x \pmod 1$ (check it)
has $3^k - 1$ fixed pts since

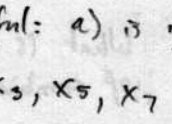


3^k sloping lines $\to$ $3^k - 1$ intersections w

Table:

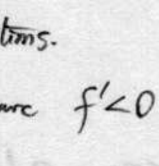
k	# f.p. of f^k	# f.p. due to factors of k	Orbits of period k
1	2	0	2
2	8	2	3
3	26	2	8
4	80	8	18.

1.2 a)



$x \in [0,1]$ has 0^+ as limit,
all other x heads to $-\infty$.

b)



0 is a source.

In a) & b) $|f(p)'| = 1$ so no info from Thm. 1.5!

c) $\arctan x$ or $\sin x$ or $x - x^3$ d) $\tan x$ viz part b).

Careful: a) is <u>neither</u> sink nor source. Use precise definitions.

1.4 b) x_3, x_5, x_7 have $f' > 0$ but x_2, x_4, x_8 have $f' < 0$. Since $(f^3)'(p_1) = (f^3)'(p_2)$ these must be the orbit groupings.

p.31 compu Expt 1.4: For most points x_0, $O(1)$ separation (e.g. $\geq 1/2$) achieved by ≈ 10 iters. for initial $\varepsilon = 10^{-3}$.
For $\varepsilon = 10^{-15}$, takes ≈ 50 iters.
Exceptions are if $x_0 = 0$ or 1, where iteration counts are only <u>half</u> the above! (due to behavior of $x = \frac{1-\cos\theta}{2}$ conjugacy map at $x = 0, 1$, making ε effectively $\sqrt{\varepsilon}$ instead)
Best plot is semilogy of absolute difference $x_n - y_n$, looks like