

MATH 53 WORKSHEET Binary fractions & shifts

Dunnett
2007

a) First integers: find $n=61$ in binary:
remainder

What is a recipe which works for any number, eg 17513? (is algorithm) (Don't do it!)

b) Fractions: What fraction is $0.\overline{101}$?

c) Find $\frac{1}{7}$ in binary: (then do $\frac{1}{9}$)

d) Show how the algorithm for getting binary expansion is the same as applying $2x \pmod{1}$ map.

e) So precisely which $x \in (0,1]$ give chaotic orbits?

SOLUTIONS

a) First integers: find $n=61$ in binary: $2^5 = 32 \leq n$
 $2^6 = 64 > n$ so $n = 1xxxxx$

remainder $61 - 32 = 29 = 16 + \text{remainder}$
 $13 = 8 + 4 + 1$ $n = 111101$

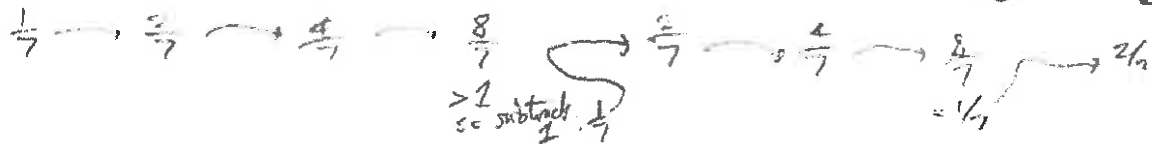
What is a recipe which works for any number, eg 17513? (Don't do it!)
 (is algorithm)

↳ find largest $2^k \leq n$, then $n \leftarrow n - 2^k$, $k \leftarrow k - 1$
 if

b) Fractions : What fraction is 0.101 ?

$$8x = 101 \cdot 101 = 5 + x \quad x = 5/7$$

d) Find $1/7$ in binary: $1/7 \xrightarrow{\text{mult by 2}} 2/7 \xrightarrow{\text{mult by 2}} 4/7 \xrightarrow{\text{mult by 2}} 8/7$
 $8/7 > 1$ so put "1"
 $8/7 - 1 = 1/7$



0 . 0 0 1 0 0 1 0 0 ...

d) Show how the algorithm for getting binary expansion is the same as applying $2x \pmod{1}$ map. See above.

if $x < 1/2$ then the "mod 1" not used, write "0" since binary for x is $x \geq 1/2$ then "is" " " "1" 0.1..... 0.0...

e) So precisely which $x \in [0, 1]$ give chaotic orbits ?

ones with nonrepeating binary expansions, which must therefore be irrational.