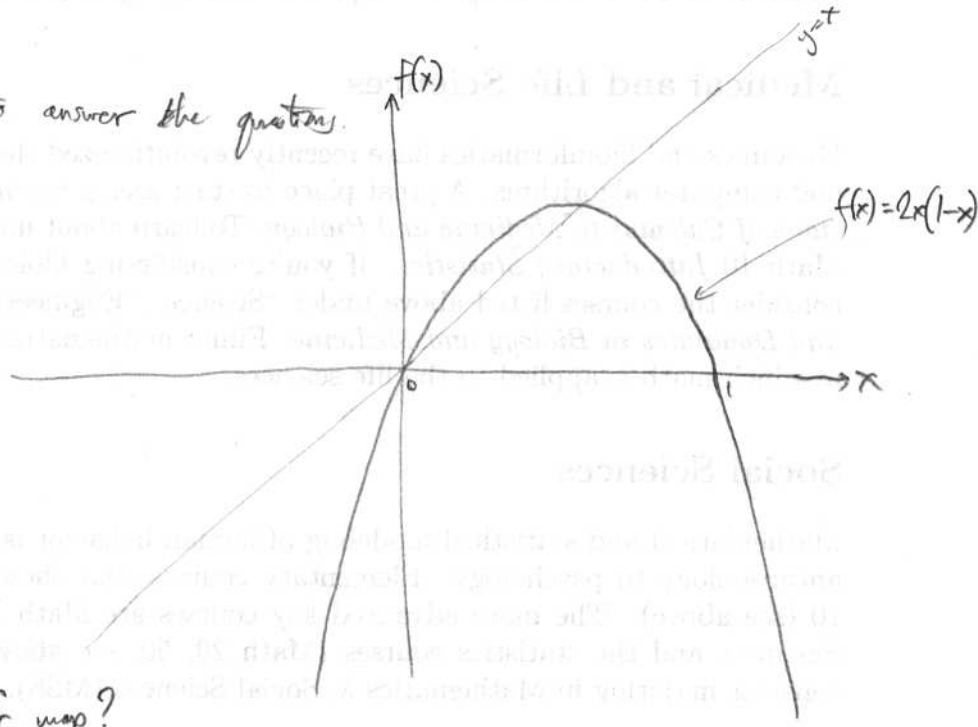
Math 53 WORKSHEET: iteration & cobweb plots. 9/26/07

A) If $f(x) = 2x(1-x)$

Find f^2: (simplify to a polynomial)

f^3: (don't bother to simplify!) Messy isn't it?

B) Sketch cobweb plots here to answer the questions.



i) Where are fixed point(s) of map?

ii) Say $x_0 = 0.1$, where does iteration take you?

Same for $x_0 = -0.1$

$x_0 = 0.9$

$x_0 = 1.1$

iii) Which fixed point is attracting? repelling?

iv) Find the basin of the attracting fixed point, ie set of all x_0 that have it as limit

v) Same for repelling fixed pt.

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SOLUTIONS

A) If $f(x) = 2x(1-x) = 2(x - x^2)$

find $f^2(x) = 2\left(2(x-x^2) - 2^2(x-x^2)^2\right)$ (simplify to a polynomial)

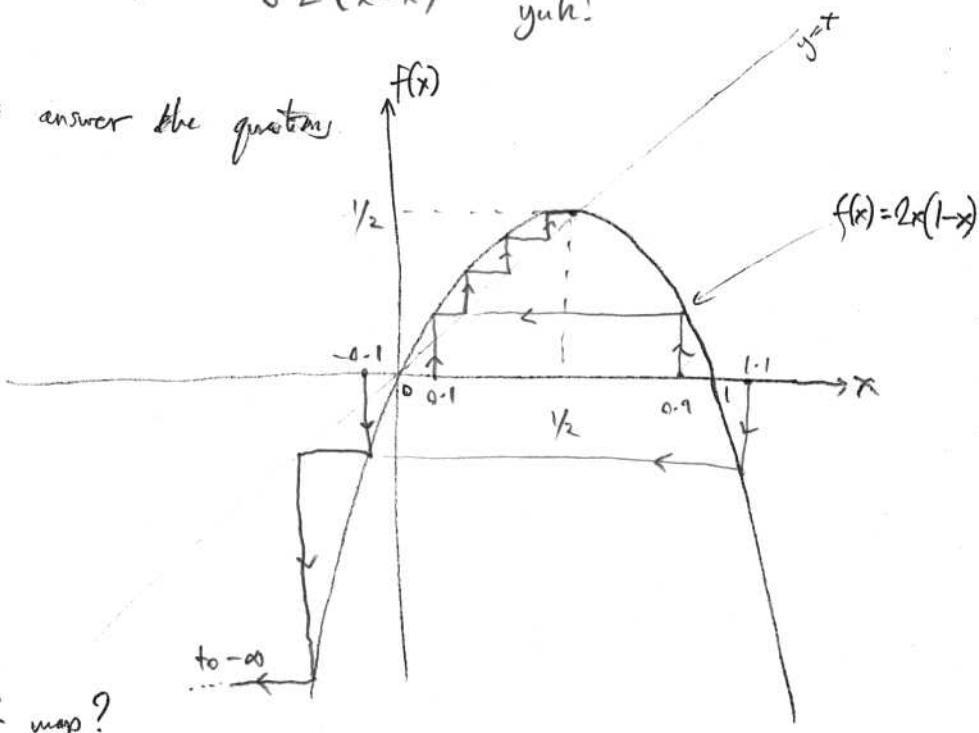
$= 4x - 4x^2 - 8x^2 + 16x^3 - 8x^4$

$-12x^2$

$f^3(x) = 4\cdot 2(x-x^2) - 12\cdot 2^2(x-x^2)^2 + 16\cdot 2^3(x-x^2)^3 - 8\cdot 2^4(x-x^2)^4$ (don't bother to simplify!)

Messy isn't it? yuh!

B) Sketch cobweb plots here to answer the questions



i) Where are fixed point(s) of map? Where graph intersects $y=x$, ie $x=0$, $x=1/2$

ii) Say $x_0 = 0.1$, where does iteration (repeated!) take you? approaches $x=1/2$

Same for $x_0 = -0.1$ to $-\infty$

$x_0 = 0.9$ joins the orbit of 0.1, so $\longrightarrow$ $x=1/2$

$x_0 = 1.1$ joins the orbit of -0.1, so $\longrightarrow$ $-\infty$.

iii) Which fixed point is attracting? ($x=1/2$) repelling? $x=0$

iv) Find the basin of the attracting fixed point, ie set of all x_0 that have it as limit. $\hookrightarrow (0,1)$

v) Same for repelling fixed pt. $\{0, 1\}$, just 2 points.

The basin of infinity is $(-\infty, 0) \cup (1, \infty)$.