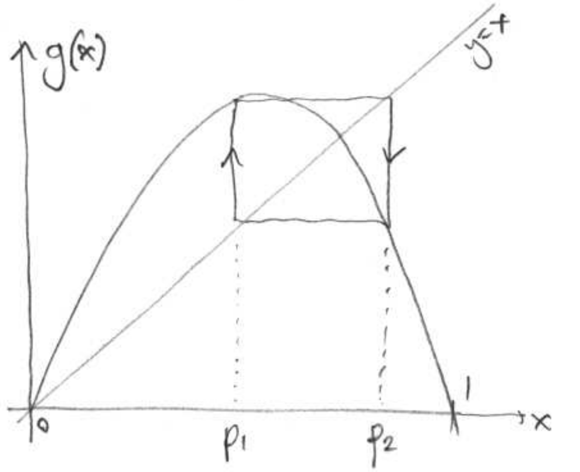
Math 53 WORKSHEET : Periodic sinks & sources. 9/28/07



Consider $g(x) = \frac{7}{2}\, x\,(1-x)$

ie logistic map with $a = 7/2$

I'll tell you $x = 3/7$ is a fixed point of g^2.

a) Is there a period-2 orbit? If so, what? $\{\underset{\uparrow p_1}{\cdot}\ ,\ \underset{\uparrow p_2}{\cdot}\}$

b) How many fixed points does g^2 have, at least?

c) Is $p_1 = 3/7$ a periodic sink/source/can't tell of period 2?

d) Is p_2 also a period-2 sink/source/can't tell?

Does this answer agree with p_1? Explain.

e) Generalize the derivative test: if $\{p_1, p_2, \cdots p_k\}$ is a period-k orbit of f,

What is $(f^k)'$ at $x = p_1$, in terms of f'? [Hint: induction ...?].

Does the test care which of $p_1, p_2, \cdots p_k$ you evaluate $(f^k)'$ at?

Why?

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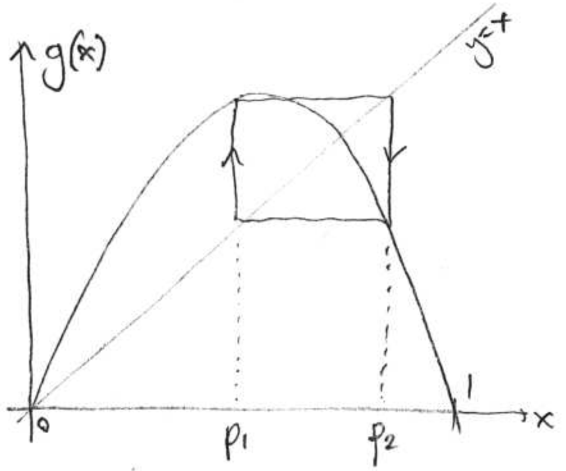
— SOLUTIONS —



Consider $g(x) = \frac{7}{2} x (1-x)$

ie logistic map with $a = 7/2$

I'll tell you $x = 3/7$ is a fixed point of g^2.

a) Is there a period-2 orbit? If so, what? $\{\underset{\uparrow p_1}{3/7}, \underset{\uparrow p_2}{6/7}\}$

Yes since $g(3/7) = 6/7$, $g(6/7) = 3/7$

b) How many fixed points does g^2 have, at least? 4 since 2 from the period-2, and 1 from each fixed pt of g

c) Is $p_1 = 3/7$ a periodic sink/source/can't tell of period 2?

$$|(g^2)'(p_1)| = |g'(p_1)\,g'(p_2)| = \left|\left(\tfrac{7}{2} - 7\cdot\tfrac{3}{7}\right)\left(\tfrac{7}{2} - 7\cdot\tfrac{6}{7}\right)\right| = \left|\tfrac{1}{2}\cdot\tfrac{-5}{2}\right| = \tfrac{5}{4} > 1$$

use $g'(x) = a(1-2x) = \left(\frac{7}{2} - 7x\right)$

a source

d) Is p_2 also a period-2 sink/source/can't tell? Yes since $|g'(p_2)\,g'(p_1)| = |g'(p_1)\,g'(p_2)|$

Does this answer agree with p_1? Explain.

yes since multiplication doesn't care about order.

e) Generalize the derivative test: if $\{p_1, p_2, \cdots p_k\}$ is a period-k orbit of f,

What is $(f^k)'$ at $x = p_1$, in terms of f'? [Hint: induction ...?].

$(f^3)'(p_1) = f'(p_1)(f^2)'(p_2) = f'(p_1)f'(p_2)f'(p_1)$. Etc. keep repeating:

$$(f^k)' = \prod_{j=1}^{k} f'(p_j) = f'(p_1)\cdots f'(p$$

Does the test care which of $p_1, p_2, \cdots p_k$ you evaluate $(f^k)'$ at?

Why? No, since you end up with the same k factors, just in different cyclic order.