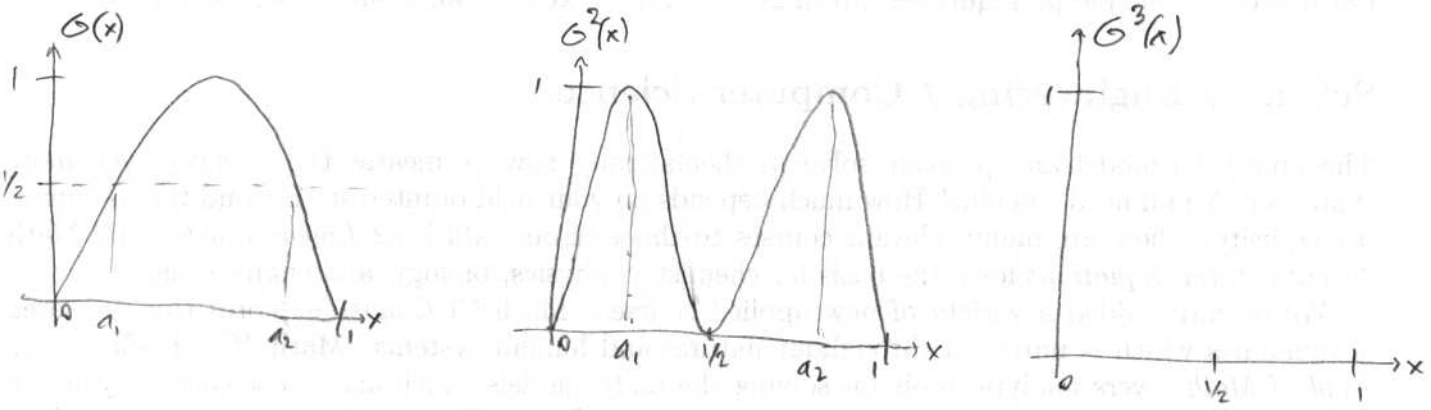
MATH 53 WORKSHEET : Counting periodic orbits.

19/1/07
Barnett



A) Continue argument to sketch G^3. How many fixed points of G^3 must there be?

B) How many fixed points of G^3 are accounted for by ...
period-1 orbits ?
period-2 orbits ?
[Hint: which lower periods give fixed pts of G^3 ?]

C) So, how many period-3 orbits are there ?

D) How many fixed points of G^k, $k \geq 1$, are there ? (proof?)

E) Continue this 'periodic table' :

period k	num. fixed pts of G^k	num. fixed pts. due to lower periodic orbits	num. orbits period k
1	2	0	2
2	4	2 period-1 orbits = 2	1
3			
4			
5			
6			
7			
⋮			

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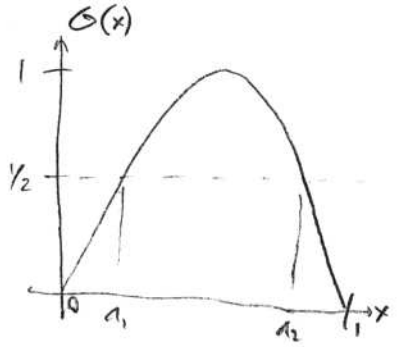
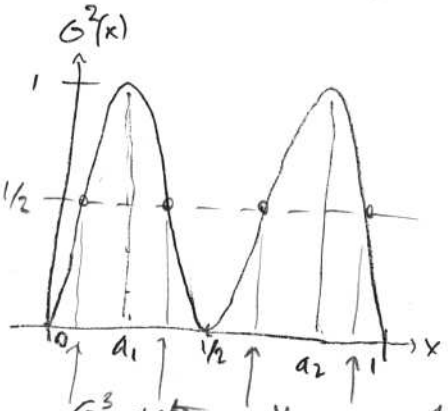
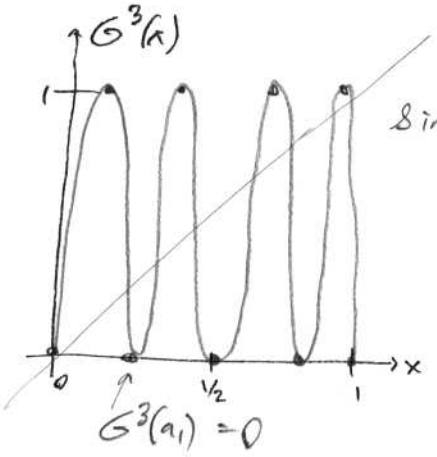
— SOLUTIONS —







A) Continue argument to sketch G^3. How many fixed points of G^3 must there be? 8.

B) How many fixed points of G^3 are accounted for by ...

period-1 orbits ? 2.

period-2 orbits ? 0. since $G^3(p_1) \neq p_1$ if $\{p_1, p_2\}$ a period-2.

$p_1 \xrightarrow{G} p_2 \xrightarrow{G} p_1 \xrightarrow{G} p_2$ (G^3)

[Hint: which lower periods give fixed pts of G^3 ?]

C) So, how many period-3 orbits are there? $8 - 2 = 6$ due to $p\text{-}3 = 2$

D) How many fixed points of G^k, $k \geq 1$, are there? (prove?) 2^k ↑ fixed points thru

E) Continue this 'periodic table':

period k	num. fixed pts of G^k	num. fixed pts. due to lower periodic orbits	num. orbits period k
1	2	0	2
2	4	2 period-1 orbit = 2	1
3	8	2	2
4	16	2 p-1 + 1 p-2 = 4	3
5	32	2	30/5 → 6
6	64	2 + 6 + 2 = 10	54/6 → 9
7	128	2	126/7 → 18
⋮		the	