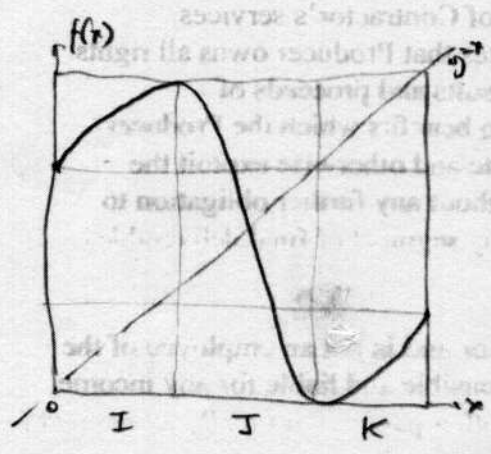


Consider the following map $(f \text{ monotonic in each interval } I, J, K)$



Draw the transition graph above.

Hint: does $f(I) \supset I$?
 $f(I) \supset K$? etc...

Prove there's a fixed point of f in J :

Prove there's a fixed pt of f^2 with $p_1 \in I, p_2 \in K$:

Categorize all possible infinite sequences of symbols:
is \overline{KI} legal? What if start with J ?

Prove that the periodic orbits of f have period 1 or 2, no others

- i) Explain why no odd periods 3, 5, ... possible in I or K :
- ii) Check f^{2k+1} monotonically decreasing in J , then explain why no odd periods ≥ 1 in J .
- iii) [BONUS] Check f^2 monot. increasing in I (or K), so $x_1, x_2 \in I, x_1 > x_2 \Rightarrow f^2(x_1) > f^2(x_2)$.

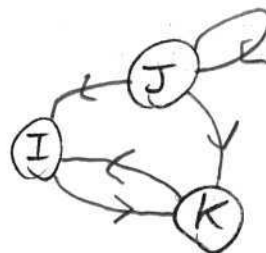
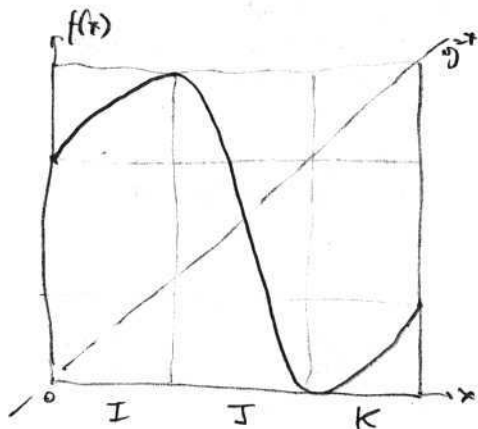
Explain why rules out even periods > 2 :

Do same for f^2 in J ; even periods

SOLUTIONS

Consider the following map

[see T3-11]



Note: $J \rightarrow I$ means $f(J) \supset I$, not that there is no such $x \in J$ with $f(x) \in I$!

Draw the transition graph above.

Hint: does $f(I) \supset I$? \times
 $f(I) \supset K$? \checkmark etc...

Prove there's a fixed point of f in J : \bullet $f(J) \supset J$ so by Fixed Pt Thm, J fixed pt in J .

Prove there's a fixed pt of f^2 with $p_1 \in I, p_2 \in K$:

$$f^2(I) = f(f(I)) = f(K \cup \{\text{possible other intervals}\}) \supset I \text{ so}$$

Categorize all possible infinite sequences of symbols:
g is \overline{KI} legal? What if start with J ?

Proof: apply fixed pt Thm to f^2 on I .

$$\begin{aligned} &\overline{J} \\ &J^n \overline{KI} \\ &J^n \overline{IK} \end{aligned} \quad \begin{aligned} n &= 0, 1, \dots \\ n &= 0, 1, \dots \end{aligned}$$

tricky since requires monotonicity of f in I, J, K intervals.

Prove that the periodic orbits of f have period $\sqrt{1}$ or 2 , no others:
no odd POs > 1 since: i) they cannot be in I or K because $I \times I$ is not possible even number of symbols

ii) In J , f^{2k+1} is monotonic decreasing ($k=1, 3, \dots$) so $p_2 > p_1 \Rightarrow f^{2k+1}(p_2) < f^{2k+1}(p_1)$

Applying to the PO $\{p_1, \dots, p_{2k+1}\}$ leads to a contradiction since $f^{2k+1}(p_j) = p_j \quad \forall j=1 \dots 2k+1$

no even POs > 2 since f monot. incr. in both I & K , so f^2 monot. incr. in both I & K , so if $\{p_1, \dots, p_k\}$ is period- $2k$ then $f^2\{p_1, \dots, p_k\}$ preserves the ordering \Rightarrow contradiction since f^2 must permute