Update: Definition of homoclinic orbits and energy-like functions, e.g. double-well potential example. Limit sets $\omega(x_0)$ (or "forward limit sets").

1. Definitions and ideas:
   Eeventually/asymptotically-periodic, attracting/repelling fixed points/periodic orbits, sink/source/saddle, basin of attraction, sensitivity to initial conditions, dense, chaos, bifurcations, (un)stable manifolds, invariant sets, Lyapunov exponents.
   Equillibria and limit cycles/periodic cycles, stable/unstable, basin of attraction, dense, chaos, (un)stable manifolds for continuous dynamical systems, invariant set, Julia set, Mandelbrot set.
   Strange attractors. Delay-embeddings, recurrence plots, Shannon entropy, KL divergence, permutation entropy, cellular automata. Arithmetic and geometric mean. Purpose of Lotka-Volterra model.

2. Be able to talk about the two types of bifurcations and compute bifurcations. Draw (in)stability of periodic orbit diagrams (i.e. small versions of the logistic pitchfork bifurcation diagram including dashed lines for unstable periodic orbits).

3. Find periodic points and evaluate their stability in one and two-dimensional maps using linearization.

4. The doubling map, logistic map, tent map, and the Hénon map. Logistic model as it relates to population dynamics.

5. Main theorems: evaluate stability, Three Implies Chaos, Lyapunov exponent of asymptotically periodic points.

6. Be able to discuss the long term behavior of maps and the existence and number of periodic points.

7. Calculate the Lyapunov exponent of one-dimensional maps. Comfort with the difference between Lyapunov number and exponent.

8. Practice building examples and counterexamples.

9. Find stable and unstable manifolds for linear maps. Verify that a set is invariant for non-linear maps in order to calculate stable and unstable manifolds for nonliner maps.

10. Give a geometric argument to determine the basin of attraction.

11. The meaning of Jacobian for the case of non-linear two-dimensional maps and why we use it.

12. Some understanding of Poincare's struggle with the 3-body problem (included in the book).

13. Conjugacy of maps. Use this to show the existence of k-periodic points for maps. Eg. $f_4(x) = 4x(1-x)$ has similar properties as the tent map.

14. Transition graph for maps. Use the transition graph to conclude that a k-periodic point does not exist.

15. Comfort working with expansion of numbers base N as it relates to Cantor set. Be able to compute infinite sums using geometric series.

16. Compute the fractal dimension using $r^d = m$. Compute the length/area of fractals and the loose meaning of "measure zero." Compute the box-counting dimension of fractals. Eg. circles, Cantor sets, $\mathbb{Q} \cap [0,1]$ and $\{0, 1, 2, 3, 4, 5\}$.

17. Writing down iterated function systems to create a fractal. Probabilistic construction of fractals, called *the chaos game*. Definition of affine contraction mappings and what this means about the image produced by the probabilistic construction.

18. Determine the Julia set of simple maps. Test whether a point is in the Mandelbrot set.

19. Sketch the phase plane to determine long term behavior of trajectories for linear maps (or in small neighborhoods when we linearize at a point). Be able to sketch using polar coordinates. For one-dimensional differential equations, use the vector field (in this case the line) to sketch a time series for the system. Eg. $\dot{x} = \sin x$.

20. Find the general solution of coupled linear systems of differential equations $\dot{v} = Av$. Use this to find the stable/unstable manifolds of linear systems.

21. Find equillibria of nonlinear systems. Determine stability of equillibria of non-linear systems of differential equations using linearization.

22. Use the Poincaré-Bendixson Theorem to show the existence of a stable limit cycle.

23. Be able to compute/discuss the Shannon entropy of a random variable X (a set of states with a probability distribution). Be able to compute/discuss the KL divergence of X from a model Z.

24. General idea of Lorenz attractor. What is it? What does it mean?
25. Sensitivity to initial conditions and predictability.
26. How might you measure how unpredictable a time series is? Why?
27. Be able to plot recurrence plots and interpret them to some extent. Periodic behavior, iterated maps with low/high sensitivity to initial conditions.
28. Be able to model cellular automata; use the rules to determine long-term behavior.
29. Distinction between randomness and chaos.