Overview: In ecology, a discrete time model is particularly applicable when generations do not overlap, that is, only one breeding generation present at each time. This property occurs in insect populations and is an assumption that is sometimes made while studying population dynamics (even when there is some degree overlapping generations).

Let x_t be the number of breeding insects at time t. We may suppose that the number of offspring, which themselves will enter the breeding generation at time $(t+1)$, only depends on the number of insects in the breeding generation in time t. Therefore, there is some function f such that

$$x_{t+1} = f(x_t).$$

On the first day of class, we talked about why the logistic map

$$f_{r,k}(x) = rx(1 - \frac{x}{k}),$$

is often taken to be a reasonable model. In particular, small populations grow roughly proportional to their size and populations that are too large collapse in the next generation. We have seen that the dynamics of the logistic map are far from simple, and depending on the value of $r \in (1, 4)$, we may have oscillating or chaotic behavior.

Suppose that we have an insect population that both evades long-term predictions (i.e. it does not appear to fall into a periodic oscillation nor a steady state) and varies wildly from year to year. I have attached the time series data for this insect population on our course webpage. Environmentalists worry about the potential extinction of the species following one of these dramatic crashes, and about the effects of an outbreak on the rest of the ecosystem.

a) Find the logistic map that best approximates the behavior of the system. Does the dynamical system defined by iterating the logistic map with this parameter appear to be chaotic? Simulate the system using cobweb diagrams to search for any attracting periodic orbits.

b) Estimate the average size of the population in this model by computing the average value and standard deviation of the iterates $f^t(x_0)$ for $0 \leq t \leq N$ and a randomly chosen x_0. How does this compare to the actual average size and standard deviation of the population as obtained from the data?

Ecologists propose a method of population control that uses pesticide to keep the population of insects to be no greater than some level, M. More concretely, $x_{t+1} = \min\{f(x_t), M\}$.

c) Simulate the model for several (at least 6) different values of M and present your findings. In a table, report long term behavior, mean and standard deviation of the population. Do you have any explanation for the (perhaps initially surprising) consequences of these methods?

d) Since it is unlikely that population control measures will prune the population to exactly M, modify your model to include noise. You can modify a function, f, to include normally distributed noise with mean $\mu = 0$ and standard deviation $\sigma = 1$ by
`fnoise[x_]:=f[x] + RandomVariate[NormalDistribution[0, 1]].`

Change the values of these errors to levels that you think are biologically meaningful in this context. Choose a few different levels of noise and simulate the long-term behavior and average population in these cases and add this information to your table from the previous question.

e) Finally, write a short paragraph justifying the choice of M that you think the ecologists should use for control of chaotic insect populations.