Let $K_0 = \{(0, y) : 0 \leq y \leq 1\}$. Define affine maps f_1, f_2, f_3 of $\mathbb{R}^2$ so that $f_1(K_0)$ is the trunk of the tree, $f_2(K_0)$ is the left branch and $f_3(K_0)$ is the right branch.

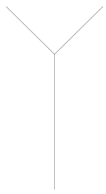

Figure 1: The length of the branches is $r = \frac{1}{2}$ and the angle is $\theta = \pi/4$.

For $j = 1, 2, 3$, if you use

```
f[j][{x_, y_}]:= {{a, b}, {c, d}}.{x, y} + {e, f}
```

to define your function, the expression `Table[Map[f[i], K0], {i, 1, 3}]` will apply each of the functions f_i to $K_0 = \{\{0, 0\}, \{0, 1\}\}$ and store it in a list. Your output is K_1 and should be of the form

```
{{{0, 0}, {0, 1}}, {{0, 1}, {x1, y1}}, {{0, 1}, {x2, y2}}}.
```

Use `Graphics[{Line[K1]}]` to plot the collection of lines whose endpoints are in K_1; which should look like the tree sketched above. Apply this procedure iteratively to obtain and plot K_i for $1 \leq i \leq 7$ (after about $i = 10$, computations become unfeasible). In your code make that you are not accidentally calling a difficult computation repeatedly since this will slow your code down considerably. I found it useful to calculate $K_1, K_2, K_3, \ldots$ one at a time, using the result from the previous computation to calculate the next. At the second iteration, I used

```
Flatten[Table[Map[f[j], K1[[i]]], {i, 1, 3}, {j, 1, 3}], 1]
```

to apply the map to each pair of endpoints. (You may find a better way.) Modify your code to allow you produce trees with a variety of values for r and θ.

In Question 4 you will need to modify your code to play the chaos game. The following syntax may help.

```
MapRandom[{x_, y_}] := f[RandomChoice[{.333, .333, .333} -> {1,
                            2, 3}]][{x, y}]
     ListIterates = NestList[MapRandom, {x0, y0}, 1000000]
                  Graphics[Point[ListIterates]].
```

Questions

1. For a fixed θ, as r is increased, the tree will eventually intersect itself. The *critical value* of a tree with angle θ is the minimal value of r for which the tree with parameters r and θ is self-intersecting. Estimate $r_c(\theta)$ for $\theta \in \{\frac{\pi}{3}, \frac{pi}{2}, \frac{3\pi}{5}, \frac{5\pi}{6}\}$.

2. For $\theta = \pi/3$ and $r_c(\pi/3)$, what does the outline of the top of the tree remind you of?

3. Compute the fractal dimension of the outermost layer of the non-self-intersecting trees. Use the simple definition with scaling factor.

4. Is f_1, f_2, f_3 an affine contraction mapping? Why or why not? Will the chaos game applied to this system produce the tree?

5. Use the code above to play the chaos game for your favorite parameters of this fractal-tree iterated function system. Remove the first 20 iterations of the map to ignore the transient behavior. What do you see? Why?

6. Implement an IFS for a system for which $K_0 = \{(0, y) : 0 \leq y \leq 6\}$ and the endpoints of the line segments in K_1 are sketched below:

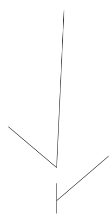

Figure 2: The endpoints of the lines are `{0, 0}, {0, 0.96}, {0., 1.6}, {0.24, 6.7}, {0., 1.6}, {-1.56, 2.92}, {0., 0.44}, {1.68, 1.88}`.

As with the Koch Snowflake, make sure that you are not changing the orientation of the branches.

7. Is this an affine contraction mapping? Play the chaos game for this fractal.

8. Design two distinctively different fractals; you may use parameters from http://paulbourke.net/fractals/ or elsewhere. Include a sketch of K_1 for each and the picture you obtain from a probabilistic construction.