

Midterm 2 Review

Past exams are a good indication of the types and difficulty of questions on the midterm. The textbook has many other good questions.

1. Definitions and ideas: equilibria and limit cycles/periodic cycles, stable/unstable, basin of attraction, dense, chaos, (un)stable manifolds for continuous dynamical systems, invariant set, Julia set, Mandelbrot set.
2. Conjugacy of maps. Use this to show the existence of k -periodic points for maps. Eg. $f_4(x) = 4x(1-x)$ has similar properties as the tent map.
3. Transition graph for maps. Use the transition graph to conclude that a k -periodic point does not exist.
4. Comfort working with expansion of numbers base N as it relates to Cantor set. Be able to compute infinite sums using geometric series.
5. Compute the fractal dimension using $r^d = m$. Compute the length/area of fractals and the loose meaning of “measure zero.” Compute the box-counting dimension of fractals. Eg. circles, Cantor sets, $\mathbb{Q} \cap [0, 1]$ and $\{0, 1, 2, 3, 4, 5\}$.
6. Writing down iterated function systems to create a fractal. Probabilistic construction of fractals, called *the chaos game*. Definition of affine contraction mappings and what this means about the image produced by the probabilistic construction.
7. Determine the Julia set of simple maps. Test whether a point is in the Mandelbrot set.
8. Sketch the phase plane to determine long term behavior of trajectories for linear maps (or in small neighborhoods when we linearize at a point). Be able to sketch using polar coordinates. For one-dimensional differential equations, use the phase plane (in this case the line) to sketch a time series for the system. Eg. $\dot{x} = \sin x$.
9. Find the general solution of coupled linear systems of differential equations $\dot{v} = Av$. Use this to find the stable/unstable manifolds of linear systems.
10. Find equilibria of nonlinear systems. Determine stability of equilibria of non-linear systems of differential equations using linearization.
11. Use the Poincaré-Bendixson Theorem to show the existence of a stable limit cycle.