1. For $n > 1$, prove that $\mathbb{R}^n$ and $\mathbb{R}$ are not homeomorphic. (Notice that this doesn't prove, for example, that $\mathbb{R}^2$ and $\mathbb{R}^3$ cannot be homeomorphic.)

2. Find (with proof) the connected components and path components of $\mathbb{R}_\ell$ and $\mathbb{R}_\ell \times \mathbb{R}$.

3. Let $GL_n(\mathbb{R}) = \{A \in M_n(\mathbb{R}) \mid \det(A) \neq 0\}$ be the set of invertible matrices.

   (a) Prove that $GL_n(\mathbb{R})$ is an open subset of $M_n(\mathbb{R})$.

   (b) Show that $GL_n(\mathbb{R})$ is not connected.[1] Determine (without proof) the connected components of $GL_n(\mathbb{R})$.

4. Determine whether the following spaces are compact or not (with proof):

   (a) The n-sphere S^n in $\mathbb{R}^n$.

   (b) $\mathbb{Q} \cap [0, 1]$.

   (c) The closed interval $[0, 1]$ in $\mathbb{R}_\ell$. (**Hint:** It's not.)

5. Let A and B be subsets of a topological space X. Prove or disprove:

   (a) If A and B are compact, then $A \cup B$ is compact.

   (b) If A is open, then A is not compact.

6. Let A and B be compact subsets of a topological space X.

   (a) Prove that if X is Hausdorff, then $A \cap B$ is compact.

   (b) By example, show that $A \cap B$ need not be compact. (**Hint:** Consider the double-headed snake $(X, \mathscr{T}_{\mathscr{B}})$ from problem 1 on the midterm and find appropriate sets A and B. To show that A and B are compact, construct a homeomorphism to a *known* compact set.)

[1] Fun fact: $GL_n(\mathbb{C})$, the invertible matrices with complex entries, is actually path connected!