1. Determine (prove or disprove) whether the following spaces are Hausdorff, T_1, or neither:
   - $\mathbb{R}_\ell$
   - $(\mathbb{R}, \mathscr{T}_f)$
   - $(\mathbb{R} \times \mathbb{R}, \mathscr{T}_{lex})$

2. Find the limit points of $A = [0, 1)$ in each of the following spaces:
   - $\mathbb{R}$
   - $(-1, 1)$ as a subspace of $\mathbb{R}$
   - $(\mathbb{R}, \mathscr{T}_d)$

3. Let $(X, \mathscr{T})$ be a topological space and $A \subset X$. Prove:
   (a) $\operatorname{Int} A$ is open in X.
   (b) $\operatorname{Bd} A = \overline{A} \cap \overline{X \setminus A}$. (Conclude that $\operatorname{Bd} A$ is closed in X.)
   (c) Let $A' = \{\text{limit points of } A\}$. Determine whether A' is (in general) an open set.

4. Consider $\mathbb{R}$ (with its usual topology). By using the interior and closure operations, we can obtain different sets. What happens when we use these operators repeatedly?
   (a) Find a set $A \subset \mathbb{R}$ so that $A, \operatorname{Cl} A,$ and $\operatorname{Int} A$ are pairwise distinct.
   (b) Find a set $A \subset \mathbb{R}$ so that we obtain 4 pairwise distinct sets by applying combinations of Int and Cl to A (e.g., A, $\operatorname{Cl} A$, $\operatorname{Int} A$, and $\operatorname{Cl} \operatorname{Int} A$).
   (c) Find a set $A \subset \mathbb{R}$ so that we obtain 5 pairwise distinct sets in this way.
   (d) (Optional/Bonus) Determine the maximum number of pairwise distinct sets that can be obtained in this way and prove it. Along the way, share an example of a set that obtains this maximum.