Are Normal Spaces Normal?

- $W = S_\Omega \times \overline{S}_\Omega$ is an open subset of the normal space $\overline{S}_\Omega \times \overline{S}_\Omega$ which is not normal.
- Hence it is another example of a regular space that is not normal.
- It is not hard to see that closed subspaces of normal spaces are normal.
- A space in which every subspace is normal is called completely normal.

Urysohn's Lemma

Theorem (Urysohn's Lemma)

Suppose that X *is normal and that* A *and* B *are disjoint closed subsets of* X*. Then there is a continuous function* $f : X \to [0,1] \subset \mathbf{R}$ *such that* $f(x) = 0$ *if* $x \in A$ *and* $f(x) = 1$ *if* $x \in B$*.*

Completely Regular Spaces

Definition

If A and B are disjoint subsets of a space X and there is a continuous function $f : X \to [0, 1]$ such that $f(A) = \{ 0 \}$ and $f(B) = \{ 1 \}$, then we say that A and B can be separated by a continuous function.

Definition

We say that a space X is completely regular if given a closed subset A and a point $x_0 \notin A$, then $\{ x_0 \}$ and A can be separated by a continuous function.

Remark

Urysohn's Lemma guarantees that normal spaces are completely regular.

Today's Work

Theorem

Subspaces of completely regular spaces are completely regular as are arbitrary products of completely regular spaces.