Closed Sets

Definition

If X is a topological space, then $F \subset X$ is said to be closed if its complement, $F^c = X \setminus F$ is open.

Theorem

Suppose that X is a topological space.

1. *Then $\emptyset$ and X are both closed.*
2. *If F_α is closed for all $\alpha \in A$, then $\bigcap_\alpha F_\alpha$ is also closed.*
3. *If F_i is closed for $i = 1, 2, \ldots, n$, then $\bigcup_{i=1}^{n} F_i$ is closed.*

Interior and Closure

Definition

Let A be a subset of a topological space X.

1. The interior of A (in X) is

$$\operatorname{int}(A) = \bigcup \{ U \subset X : U \text{ is open in } X \text{ and } A \subset U \}.$$

2. The closure of A (in X) is

$$\overline{A} = \bigcap \{ F \subset X : F \text{ is closed in } X \text{ and } A \subset F \}.$$

Remark

The interior of A is the largest open subset of A and the closure of A is the smallest closed set containing A.

Useful Terminology

Definition

If X is a topological space and $x \in X$, then an open set in X containing x is called a neighborhood of x.

Definition

If A and B are subsets of X, then we say that A meets B if $A \cap B \neq \emptyset$.

The Closure

Theorem

Suppose that A is a subset of a topological space X.

1. *We have $x \in \overline{A}$ if and only if every neighborhood of x meets A.*
2. *Suppose that β is a basis for the topology on X. Then $x \in \overline{A}$ if and only if given $U \in \beta$ with $x \in U$, we have $U \cap A \neq \emptyset$.*