The Ordered Square

Theorem

The ordered square I_0^2 is a linear continuium. In particular, I_0^2 is connected.

Path Connected Spaces

Definition

If X is a topological space, then a path in X from x to y is a continuous function

$$f : [a, b] \subset \mathbf{R} \to X$$

such that $f(a) = x$ and $f(b) = y$. We say that X is path connected if every pair of points in X is joined by a path in X.

Lemma

Path connected spaces are connected.

Bad Apples

Example

The ordered square I_0^2 is connected but not path connected.

Example

The topologist's Sine Curve is

$$\overline{S} = \{ (x, \sin(\tfrac{1}{x})) \in \mathbf{R}^2 : x \in (0, 1] \} \cup \{ 0 \} \times [-1, 1].$$

Then $\overline{S}$ is connected by not path connected.

Compactness

Definition

A topological space X is called compact if every open cover of X has a finite subcover.

Lemma

Let K be a subspace of X. Then K is compact if and only if every open cover of K in X has a finite subcover.