A Cute Corollary

Definition

A point x is a topological space is said to be isolated if $\{ x \}$ is open.

Theorem

Suppose that X is a nonempty compact Hausdorff space without isolated points. Then X is uncountable.

Corollary

The reals are uncountable. In fact, any interval or ray in $\mathbf{R}$ is uncountable.

Limit Point Compactness

Definition

A topological space X is called limit point compact if every infinite subset of X has a limit point in X.

Theorem

Every compact space X is limit point compact, but the converse can fail.

Definition

A topological space X is called sequentially compact if every sequence in X has a convergent subsequence in X.