Homeomorphisms

Definition

Suppose that X and Y are topological spaces. Then a bijection $f : X \to Y$ is called a homeomorphism if both $f : X \to Y$ and $f^{-1} : Y \to X$ are continuous.

Remark

A map $f : X \to Y$ is a homeomorphism if

1. f is a bijection,
2. $f^{-1}(V)$ is open in X if Y is open in Y, and
3. $f(U)$ is open in Y if U is open in X.

Remark

A continuous bijection need not be a homeomorphism.

Simple Products

We saw that there were lots of ways to construct continuous functions in general situations. I won't repeat Theorem 18.2 or the Pasting Lemma (Theorem 18.3) here, but it is worth reviewing them.

Theorem

Suppose that X, Y, and Z are topological spaces and that $f : Z \to X \times Y$ is a function given by $f(z) = \big(g(z), h(z)\big)$ for $g : Z \to X$ and $h : Z \to Y$. If $X \times Y$ has the product topology, then f is continuous if and only if both g and h are.