Math 56 Compu & Expt Math, Spring 2013: Quiz 2

in X-hr 5/8/13, 35 mins, just pencil and paper

1. (a) Compute the Fourier series coefficients $\hat{f}_m$ for

$$f(x) = \begin{cases} 1, & 0 \le x < \pi, \\ 0, & \pi \le x < 2\pi. \end{cases}$$

Your answer shouldn't involve any exponentials.

(b) Compute the sum of the squared magnitudes of the Fourier coefficients [Hint: don't use (a)].

(c) Is it possible that there is a set of complex numbers $\{d_m\}_{|m|<10}$ such that $\|\sum_{|m|<10} d_m e^{imx} - f(x)\|$ is smaller than $\|\sum_{|m|<10} \hat{f}_m e^{imx} - f(x)\|$?

2. What interpolant function is produced if $N = 4$ point trigonometric polynomial interpolation is carried out on the function e^{i7x}, on the usual interval $(0, 2\pi)$?

   [BONUS] How many sample points are needed so that the previous function is interpolated exactly?

3. A function f has Fourier coefficients bounded by $\hat{f}_m \leq r^{|m|}$ for some $r < 1$. As $N \to \infty$, what type of convergence do you expect in the error of the mth Fourier mode $\hat{f}_m$ when it is approximated by $\tilde{f}_m$, i.e. via the DFT of N samples?

   Derive a *rigorous* upper bound on this error (you may assume N is sufficiently big):

4. Compute the result when the vector [1 2 3] is acyclically convolved with the vector [3 2 1].

5. Roughly how many times faster would you expect Strassen's algorithm to multiply two numbers of length 10^6 digits to run than the standard long multiplication algorithm? (Explain.)