4/18/13
Barnett

MATH 56 WORKSHEET : DFT basics

$$\boxed{F_{mj} = \omega^{-mj}, \qquad \omega = e^{2\pi i/N}}$$

A) Write out F for $N=4$:

[Hint: what is ω?]

$$F = \begin{bmatrix} \; & \; & \; & \; \\ \; & \; & \; & \; \\ \; & \; & \; & \; \\ \; & \; & \; & \; \end{bmatrix}$$

Use this to compute the DFT of:

i) $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ ← const signal.

ii) $\begin{bmatrix} 1 \\ i \\ -1 \\ -i \end{bmatrix}$ ← frequency-1 complex exponential

B) Compute $\displaystyle\sum_{j=0}^{N-1} \omega^{kj}$, which is a function of k, for the cases:

$k = 0$:

$k = mN$:

$k \neq$ multiple of N : [Hint: geom series]

Conclude: "Sum Lemma"

$$\sum_{j=0}^{N-1} \omega^{kj} = \begin{cases} \quad , & k = 0 \pmod N \\ \quad , & \text{otherwise} \end{cases}$$

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updated,

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→ SOLUTIONS on... $\boxed{F_{mj} = \omega^{-mj}, \quad \omega = e^{\frac{2\pi i}{N}}}$

A) Write out F for $N = 4$:

[Hint: what is ω ?]
$\hookrightarrow \omega = e^{\frac{i\pi}{2}} = i$

$$F = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix}$$

Use this to compute the DFT of:

i) $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \xrightarrow[F]{\text{mult by}} \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ ie $\hat{f}_0 = 4$ others zero

const signal.

ii) $\begin{bmatrix} 1 \\ i \\ -1 \\ -i \end{bmatrix} \xrightarrow[F]{\text{mult by}} \begin{bmatrix} 0 \\ 4 \\ 0 \\ 0 \end{bmatrix}$ ← note only the "1" entry is nonzero.

frequency-1 complex exponential

B) Compute $\sum_{j=0}^{N-1} \omega^{kj}$, which is a function of k, for the cases:

$k = 0$: $\sum_{j=0}^{N-1} 1 = N$

$k = mN$: $\sum_{j=0}^{N-1} \omega^{Nmj} = N.$ $\quad \omega^{Nmj} \longrightarrow (\omega^N)^{mj} = 1^{mj} = 1$

$k \neq$ multiple of N : (Hint: geom series) → with $r = \omega^k$

$= \frac{1-(\omega^k)^N}{1-\omega^k}$ but $(\omega^k)^N = (\omega^N)^k = 1^k = 1$ so it vanishes: $\frac{\cancel{1-1}}{1-\omega^k}$

Conclude: "Sum Lemma"

$$\sum_{j=0}^{N-1} \omega^{kj} = \begin{cases} N, & k = 0 \pmod N \\ 0, & \text{otherwise} \end{cases}$$