

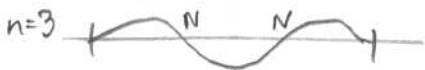
# MATHS

## Summary of string instrument timbre



Bunnett  
5/4/07

### MODES :



∴ etc.

shape functions

$$y_1(x) = \sin\left(\pi \frac{x}{L}\right)$$

$$y_2(x) = \sin\left(2\pi \frac{x}{L}\right)$$

∴

$$y_n(x) = \sin\left(n\pi \frac{x}{L}\right)$$

natural freqs.

$$\frac{c_{\text{string}}}{2L}$$

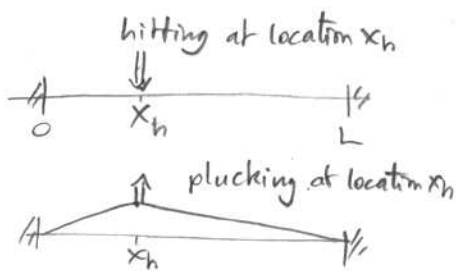
$$2 \cdot \frac{c_{\text{string}}}{2L}$$

∴

$$f_n = n \frac{c_{\text{string}}}{2L}$$

general motion is a sum of these with strengths  $\alpha_1, \alpha_2, \dots$

### EXCITATION :



[or, bowing, complicated, excites all  $\alpha_n$ ]

Basic rule :

excitation  $\alpha_n$  is zero if mode  $n$  has a node at  $x_h$

More mathematically :

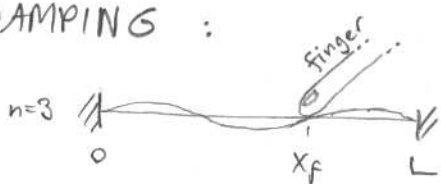
$$\alpha_n = y_n(x_h) = \sin\left(n\pi \frac{x_h}{L}\right)$$

More advanced treatment :

$$\alpha_n = \begin{cases} \frac{1}{n} \sin\left(n\pi \frac{x_h}{L}\right) & \text{hit} \\ \frac{1}{n^2} \sin\left(n\pi \frac{x_h}{L}\right) & \text{pluck} \end{cases}$$

message: plucking excites high harmonics less than hitting.

### DAMPING :



guitar/violinists call this a 'harmonic'.

lightly touching string 'kills' (damps, i.e. removes) certain modes

Basic rule :

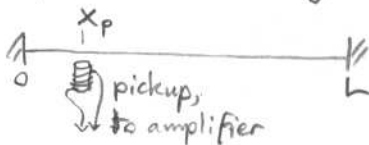
mode  $n$  survives only if there is a node at  $x_f$

$$\text{Eg } x_f = \frac{2}{3}L$$

only modes surviving are 3, 6, 9, ...

### SOUND PRODUCTION :

Exception is electric guitar →



Basic rule :

you hear the excitation  $\alpha_n$  unless mode  $n$  has a node at  $x_p$ .

Unless told otherwise, assume the partials you hear are proportional to excitations  $\alpha_n$

More mathematically :

$$c_n = \alpha_n \sin\left(n\pi \frac{x_p}{L}\right)$$

amplitude