Math 5: Music and Sound. Homework 4

due Wed Oct 20 . . . but best if do relevant questions after each lecture

1. Imagine a musician was playing an A4 (440 Hz). What frequency, and musical pitch, would you hear in the following (admittedly unlikely) situations?
   (a) You're on the ground while the musician is in a (silent!) jet plane moving at 170 m/s (380 mph) towards you.
   (b) The same except moving away from you.
   (c) The musician is on the ground and you're in the plane moving at 170 m/s towards the musician.

   BONUS: If the musician played actual music, would the *speed* (tempo) of the music change as well as the pitch? [Use the spacetime diagram for the Doppler shift to figure this out]

2. In class we discussed that the Doppler frequency shift is small for everyday speeds. Here we explore that a bit more.
   (a) A siren on an ambulance traveling at 43 miles per hour[1] towards you emits a C5 note. What note will you hear?
   (b) Repeat the above except if the siren is fixed and you're driving towards it at 43 mph.
   (c) Looking at the two above answers we notice the following approximate rule for small speeds ($v \ll c$): for every 1% of the speed of sound the relative motion is (regardless of who's fixed or moving) you get a 1% change in frequency (nice and simple!) How many *cents* does a 1% change in frequency correspond to? (Again, this helps you clarify that cents are not %)
   (d) Apply the rule to figure out if you stroll towards an orchestra at 4 mph, how many cents sharp it will appear to be.

3. Analyse the sound 'Moving train whistle' I recorded of a train blowing its whistle while moving towards me, then away from me, at some unknown speed u. You will see that the partials were shifted up and down according to the Doppler formula, but you don't know what their original frequency was. Choose one clear partial from the whistle chord, and measure carefully its frequencies f_1 and f_2 coming towards and away. Substitute each into the Doppler formula. From these two equations solve for the unknowns f and u (or, if stuck, you may simply use the average of the two frequencies as f, for slightly less credit). Thus tell me how fast the train was moving in meters per second. [Hint: This one is a bit harder, so work together. I recommend you convert the sound to WAV then use the `praat` spectrogram with 0.2 s window and 0-1000 Hz range, then measure carefully a corresponding partial in the first vs last blasts].

4. Alice (A) stands at the left (rear) end of a train, and Bob (B) at the right (front) end. The train moves to the right at a constant speed of about half the speed of sound (Japanese bullet train!). Draw a spacetime diagram showing just A and B (not the train). At $t = 0$ they clap simultaneously, and the sound travels in the fixed outside air (not inside the train—they have their heads and hands out the window!). Add their sound pulses to your diagram and use it to answer the question: do they *hear* each other's claps simultaneously? If not, who hears first and why?[2] (BONUS: If the train has length 255 m, compute the time delay between the two hearing events)

[1] 1 mph = 0.45 m/s
[2] Albert Einstein used this exact argument with pulses of *light* in his world-shattering theory of Special Relativity!

5. Download the sound `toneslouder.wav` from the website and listen to it at low volume. By using headphones and listening carefully you should be able to hear all six tones comfortably with the computer volume level fixed. (CAREFUL: it goes from very quiet to very loud so *please start at LOW volume until you've heard how loud it gets!*).

   (a) Use `audacity` to look at the graph of the signal and write down the approximate amplitudes of the six tones. (To see the first couple you will need to zoom vertically and stretch the whole track to be nice and tall).

   (b) Use `praat` to take the spectrum (not spectrogram!) of each of the tones in turn (please select the whole 1 second each time otherwise your results will vary). List the maximum heights of the six peaks—they are given in dB automatically by `praat`. What sequence do you notice?

   (c) By what factor is the *intensity* (not amplitude) in the last of the tones greater than the first? Express this factor in dB.

6. A clarinettist playing full volume radiates only about 0.05 W of acoustic power. Assume the sound comes out equally in all directions.

   (a) What is the intensity (power per unit area) reaching a listener at distance 2 meters?

   (b) What is this intensity in dB? (use the standard reference $I_r = 10^{-12}$ W/m^2)

   (c) If the clarinettist is joined by 2 more playing equally loud but everything else stays the same, compute the intensity in dB.

   (d) Going back to one clarinettist, how far away do you have to go so that they are on the threshold of human hearing (0 dB)?[3]

7. A thunderstorm clap is measured to be 130 dB when the thunderstorm is 1/5 mile away (1 mile = 1608 m).

   (a) The thunderstorm moves to 10 miles away. What intensity in dB would you expect now? [Hint: use ratios]

   (b) Compute the delay in seconds between thunder and lightning for these two distances (1/5 mile and 10 miles). Assume the light is instantaneous.

8. A mass-spring oscillator has a spring strength $k = 1000$ and mass $m = 0.001$ (let's not worry about the units).

   (a) Find the natural frequency f_0

   (b) If the spring is made 4 times stronger, what is the new natural frequency and how (by what interval) has its musical pitch changed?

   (c) Going back to the original spring, if you wanted to go *down* a perfect fifth from the original musical pitch, what value of mass m would you need?

[3] I find this answer a little shocking. It just shows how much background noise there is, how are ears get bombarded, etc. Also relevant is the no-refocusing-from-China issue!