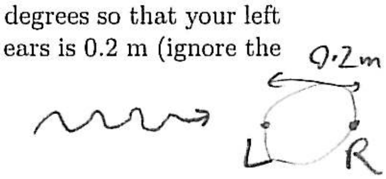
~~~ SOLUTIONS ~~~ - Barnett 10/23/08.

Math 5: Music and Sound, 2008. Midterm

2 hours, 7 questions, 60 points total

Please show working and heed the indicated number of points per question. Useful info on last page.

1. [9 points] Humans and other animals use phase difference to locate the direction a sound is coming from. Say a pure tone comes from straight ahead, and you twist your head 90 degrees so that your left ear faces the sound and your right ear faces away. The distance between your ears is 0.2 m (ignore the complication that sound has to travel around the side of your head).



   (a) What is the time delay between signals arriving at your two ears?

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$$\underset{\text{(delay)}}{\tau} = \frac{x}{c} = \frac{0.2}{340} = 0.00058 \text{ s}$$

$(5.8 \times 10^{-4} \text{ s})$

   (b) At what frequency are your ears one wavelength apart?

2

$$c = f\lambda \quad \text{so} \quad f = \frac{c}{\lambda} = \frac{340}{0.2} = 1700 \text{ Hz}$$

since $\lambda = 0.2$m. (notice: reciprocal of above τ)

   (c) Say the signal at your left ear is $\sin(400\pi t)$. What is the signal at your right ear? (ignore any amplitude change).

2

so $A = 1$ still.

$$2\pi f = \omega = 400\pi \quad \text{so} \quad f = 200 \text{ Hz}$$

so phase has size $\phi = \frac{\omega x}{c} = \frac{400\pi(0.2)}{340} = 0.739$ rad.

Right ear is delayed so phase negative: $g_R(t) = \sin(400\pi t - \phi)$

Note: you could have got by replacing t by $t - \tau$ using value from a).

   (d) Assume humans *cannot* detect a phase difference of less than 0.2 radians (about 11 degrees). For what range of frequencies are you potentially able to perceive direction? [Have you noticed this in real life?]

3

you can't tell where the thud of a bass stereo is coming from!

For what freq. is $\phi = \frac{\omega x}{c} = 0.2$ rad ?

Solve for ω: $\omega = (0.2)\frac{c}{x} = 0.2\,\frac{340}{0.2} = 340$ rad/s.

Freq. $f = \frac{\omega}{2\pi} = 54.1$ Hz.

Higher freqs cause greater ϕ phase shift so can be detected (potentially).

$\Rightarrow$ Can detect direction for freqs. ≥ 54Hz.
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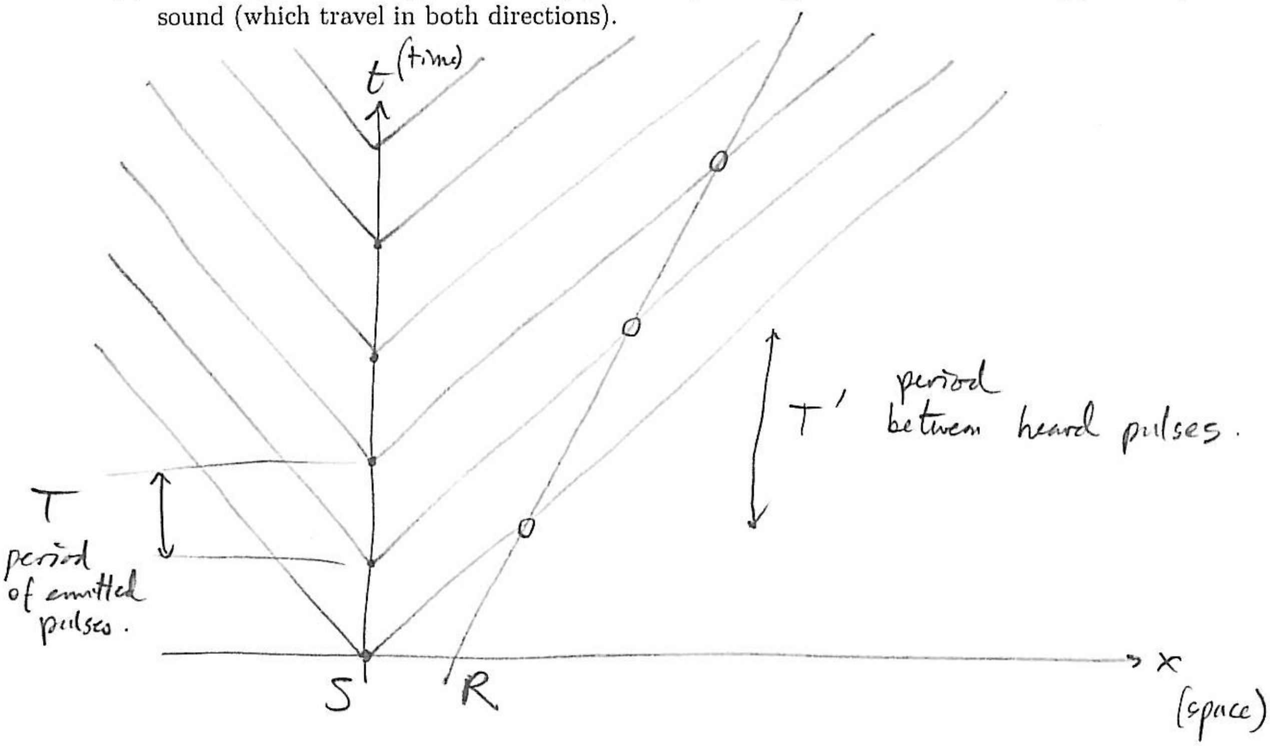
2. [7 points]

(a) Draw a spacetime diagram, labeling your axes, showing a fixed source emitting periodic pulses of sound (which travel in both directions).



(b) Now add to your diagram an observer (listener) *moving* rightwards, who starts to the right of the source. Use your diagram to explain whether they hear a frequency lower, the same as, or higher, than that of the source.

since $f = \frac{1}{T}$, and $T' > T$ from diagram.

Later pulses have to travel further ⇒ more delayed.

(c) How fast and in which direction (leftwards) would the observer have to move to hear a pitch a perfect fifth (3:2) higher than the source?

if receiver moving towards source.

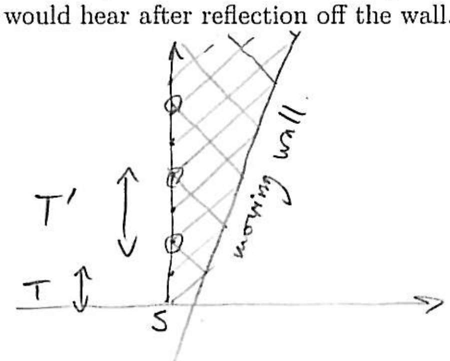
$$f_R = \left(1 + \frac{v}{c}\right) f$$

We're told $f_R = \frac{3}{2} f$

so you know receiver moves towards (leftwards).

so $\frac{3}{2} = 1 + \frac{v}{c}$, ie $v = \left(\frac{3}{2} - 1\right)c = \frac{1}{2} 340 = 170$ m/s

(d) BONUS: If you replaced the moving observer by a moving reflective wall, sketch on your diagram the new pulses and use this to say something (even a formula?) about the frequency the emitter would hear after reflection off the wall.



clearly if wall moves away (as in (b)), $T' > T$

so freq f_R is lower than f (& lower than above (b), too!)

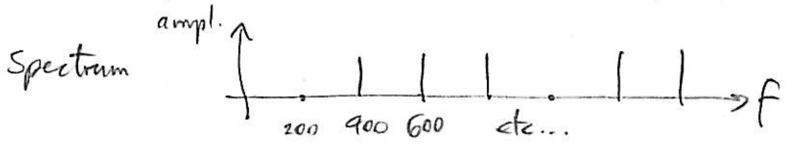
Treating the wall as a re-emitting source,

we get here $f_R = \dfrac{1 - \frac{v}{c}}{1 + \frac{v}{c}} f$

for small $v \ll c$, 'twice the effect'

3. [8 points]

(a) An instrument produces a sound whose spectrum contains the following partials (measured in Hz): 400, 600, 800, 1200, 1400. What is the (likely) perceived pitch and why?



spectrum contains only exact multiples of 200Hz, so our ear melds this together into a single tone of some timbre, at pitch 200Hz.

note: missing fundamental!

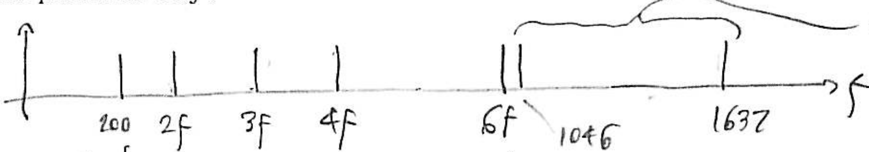
Is the signal periodic? If so, give the period. If not, explain why.

Yes, $T = \frac{1}{\text{fundamental freq}} = \frac{1}{200} = 0.005\,s$.

Harmonic series. ✓ $(\omega = 2\pi f)$.

Periodic signal $\Longleftrightarrow$ $g(t) = c_1 \sin(\omega t + \phi_1) + c_2 \sin(2\omega t + \phi_2) + c_3 \cdots$

(b) A different instrument produces instead: 200, 340, 510, 680, 1020, 1046, 1637. What is the (likely) perceived pitch and why?



these do not contribute to sense of pitch

'hum' tone of bell, maybe, but not the perceived pitch.

these 4 partials are exact multiples of 170 Hz = f

⇒ ear locks onto these & perceives strong impression of 170Hz pitch.

Is the signal periodic? If so, give the period. If not, explain why.

No, it cannot* be since period signals can contain only integer multiples of $\frac{1}{T}$, their period.

* (although strictly they are all multiples of 1 Hz, so will repeat every 1 sec. However this will have no relevance aurally since it's so much longer than individual periods. Try it: you won't hear anything repeat after 1sec!)

(c) Describe briefly the difference in sound between the two (or give examples of instruments which they could be).

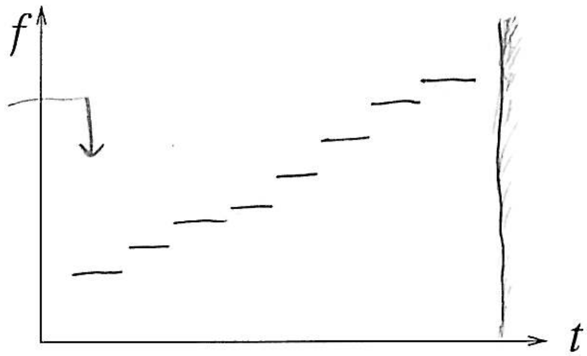
(a) : Voice, violin, etc. (any instrument producing continuous periodic signal, with significant strength in higher partials, maybe harsh)

(b) : bell, percussion instrument (since partials not all harmonically related)

4. [9 points] Sketch spectrograms on the axes provided which could realistically match the following descriptions. Feel free to highlight any features in words too:

(a) A rising musical scale played by a pure tone instrument, followed by a clap.

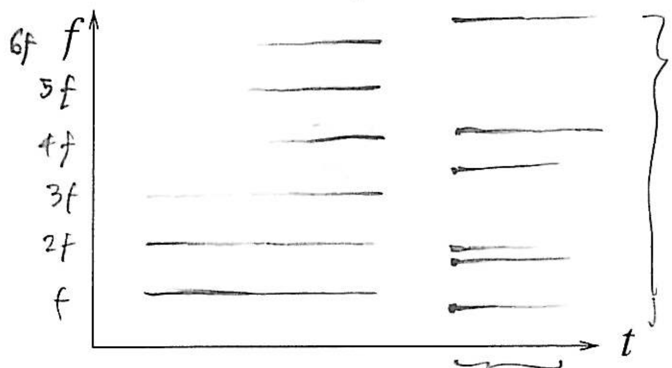
only one line



clap. (lots of high freqs, v. short in time)

(b) A singer singing a fixed pitch while changing from a mellow to a harsh timbre, followed by a struck bell of no definite pitch.

notice harmonic series since singer's note is periodic

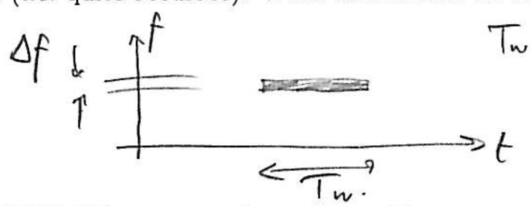


not integer-multiple partial freqs.

lots of strength in upper multiples.

probably decaying in time.

(c) Say you wanted to use a spectrogram to measure the frequency of a partial to an accuracy of 2 Hz (*i.e.* quite accurate). What restrictions on the *time window* would this place?



T_w

since $\Delta f \approx \frac{1}{T_w}$ is best freq. resolution.

you need T_w at least $1/\Delta f$, ie at least 0.5 s long.

BONUS: What types of sounds would the spectrogram then be ill-adapted to analyze?

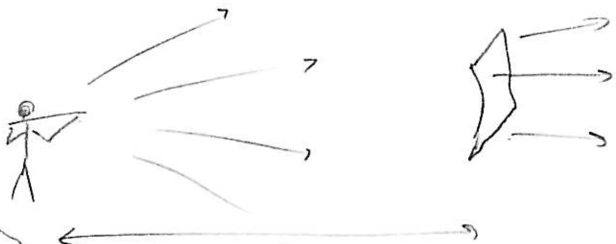
Any sounds which changed rapidly in time, eg fast notes or rapid rhythms. (faster than 0.5 s)

5. [10 points] A flute player plays a single note which can be approximated by a pure sinusoid at 1575 Hz (in this question ignore vibrato or other real-world musical complications).

(a) What equal-tempered musical pitch (give name and octave number) is this nearest, and what is the difference from this pitch in semitones?

$R = \frac{1575}{440}$ (ratio to A4) $\quad n = 12 \frac{\ln R}{\ln 2} = 22.077 = 0.077$ semitones sharp of G6 which is the note exactly 22 semitones (2 octaves minus 2 semitones) above A4.

(b) The flute player radiates 0.01 W acoustic power equally in all directions. What intensity in W/m^2 would a listener hear at a distance of 30 m?



$r = 30$ m

$$I = \frac{P}{4\pi r^2} = \frac{10^{-2}}{4\pi\, 30^2}$$
$$= 8.84 \times 10^{-7} \ W/m^2.$$

Can do quicker with ratios: $I \propto \frac{1}{r^2}$ so $I \to 10^{3} I$ needs $r \to \frac{1}{\sqrt{10^3}} r \approx \frac{30m}{\sqrt{10^3}}$

(c) The flute player now plays much quieter so that the intensity at that distance becomes 30 dB less. To what new distance from the player would the listener need to move their chair in order to hear the same loudness as they did before? → Clearly, this is closer than before.

If I changes to 30dB less, this is factor 10^{-3} intensity change (since $\frac{I_2}{I_1} = 10^{dB/10}$ for example)

Since this happened without r changing yet, P must have changed by same factor.

So, $P_{new} = 10^{-3} \cdot 10^{-2} = 10^{-5}$ W $\quad$ Now use $I_{old} = \frac{P_{new}}{4\pi r^2}$

This was a hard one

& solve for r to get $r = \sqrt{\frac{P_{new}}{4\pi I_{old}}} = 0.949$ m, ie very close!

(d) A second flute player joins the first, and you hear a single note whose amplitude maxima pulsate at 5 Hz. From this, what can you state for certain about the frequency of the second player? (This is how musicians often 'tune up' their instruments).

The two pure tones are beating, so if $f_1 = 1575$ Hz then f_2 must be ± 5 Hz different (you can't tell if it's sharp or flat merely from beat freq.)

so $f_2 = 1570$ Hz or 1580 Hz.

6. [7 points]

(a) Construct the frequency ratio from C to the A above it in the Pythagorean C major (diatonic) scale. (Briefly show working)

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$\frac{1}{2}$ down 1 octave.

C	D	E	F	G	A	B	C	D
1	9/8			3/2	27/16			9/4

$3/2$ $\quad$ $3/2$ $\quad$ $3/2$

ans: $\underline{27/16}$

(b) By how many cents does this interval differ from the equal-tempered version of the same interval?

2

C → A is major sixth (count 9 semitones on kbd).

Equal-tempered is $2^{9/12}$

Ratio between intervals is $R = \dfrac{27/16}{2^{9/12}} \approx 1.00339\cdots$

convert to cents: $c = 1200 \dfrac{\ln R}{\ln 2} = +5.9$ cents sharp of equal tempered. (pretty good!)

(c) Explain if it is possible or not to have a 12-semitone (*i.e.* Western classical) tuning system in which every perfect fourth is as the Greeks would have liked it (4:3).

1

Suppose it were possible, then $C \xrightarrow{4/3} F \xrightarrow{4/3} Bb \xrightarrow{\text{etc.}} Eb \to \cdots \text{etc} \to C$ would be a cycle of 4ths returning to C after 12 steps.

So $(4/3)^{12}$ would be some power of 2 since ~~integer~~ integer number of octaves*.

But, $(4/3)^{12} = 31.569\cdots \neq 32 = 2^5$

It is close but not exact ⇒ not possible.

*Note, if we're willing to give up the octave as exactly ratio of 2, it is possible. (but strange, probably discordant).

7. [10 points] Random short-answer questions.

(a) Write $\sin(\omega t) + \cos(\omega t)$ as a single sinusoid giving its amplitude and phase.

$A=1$ $B=1$

Right triangle gives $C = \sqrt{A^2+B^2} = \sqrt{1^2+1^2}$
$= \sqrt{2}$

$\phi = \tan^{-1}(B/A) = \tan^{-1} 1 = 45^\circ$ ($\pi/4$ rad)

$\sqrt{2}\sin(\omega t + \pi/4)$

(b) If mass is added to the prongs of a tuning fork so that its (effective) mass becomes three times larger (viewing the fork as a mass-spring oscillator), by what musical interval (up or down?) will the pitch change?

$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ so if $m \to 3m$, $f_0 \to \frac{1}{\sqrt{3}} f_0$ } by ratios

So the pitch goes down by whatever interval $\sqrt{3}$ is.

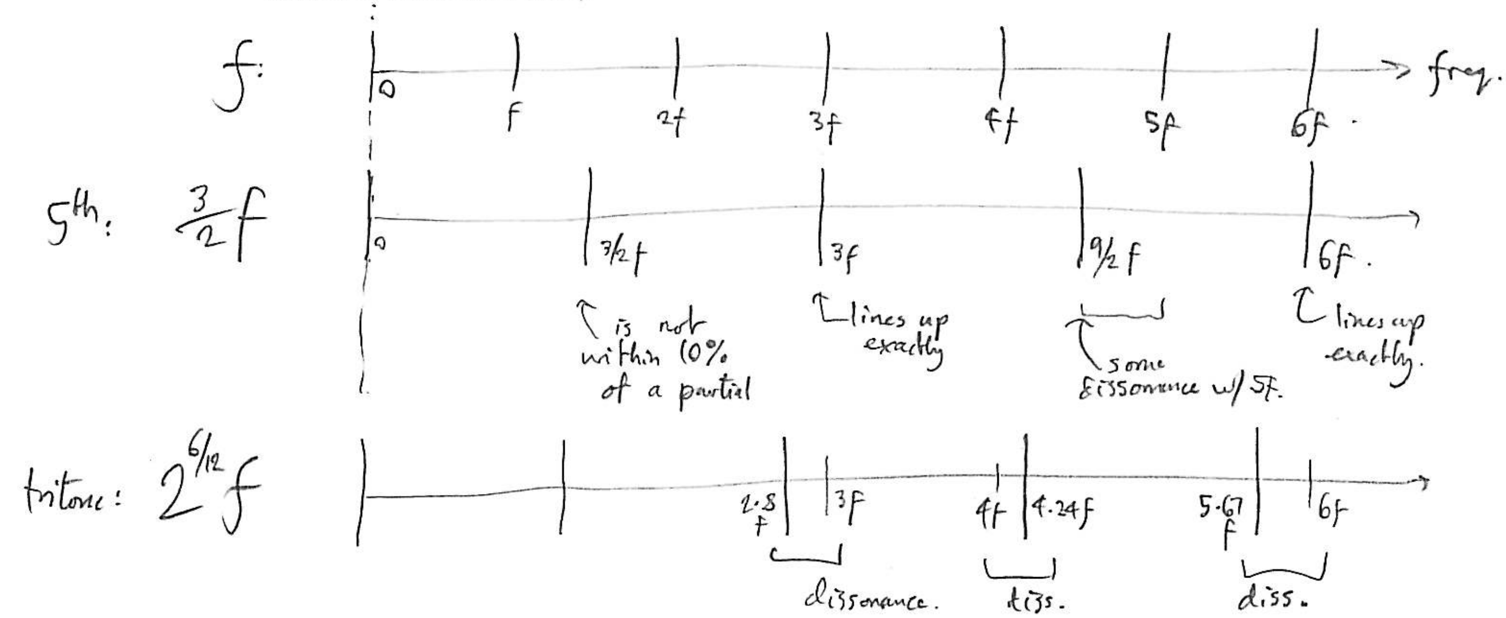
semitones $= 12 \frac{\ln \sqrt{3}}{\ln 2} \approx 9.51\ldots$ almost halfway between maj 6th & minor 7th.

(c) Compute the amplitude ratio between the quietest (0 dB) and loudest (130 dB) sounds a human can comfortably hear.

$+130\,dB = 10 \log_{10} \frac{I_1}{I_2}$ so $I_2 = \underbrace{10^{13}}_{\text{intensity ratio}} I_1$

$\Rightarrow \frac{A_2}{A_1} = \sqrt{\frac{I_2}{I_1}} = \sqrt{10^{13}} = 10^{13/2} = 3.16 \times 10^6$ big ratio!

(d) According to the Helmholtz theory, state briefly or show in a diagram why a perfect fifth (3/2, or 7 semitones) is less dissonant than a tritone (6 semitones). (Only consider up to the sixth harmonic of the lower note).



tritone has 3 dissonances but fifth only one (counting up to 6f).