MATH 5 HW5 Solutions, Fall 2010 Barnett 11/1/10.

$\boxed{1.}$ a). $g_1(t) + g_2(t) = \underbrace{2}_{\text{ampl } A=2.} \sin 2\pi f t$

If original ampl. was $A=1$ with I intensity,

since $I \propto A^2$, $A=2$ gives $4I$ intensity. (Free power!?)

2\. b). When $\theta = \pi$, out of phase and $g(t) = \sin 2\pi f t + \sin(2\pi f t + \pi)$
$= 0$

weird since where did the power go?

$A = 0$ so intensity $= 0$.

4\. c) $g(t) = \sin 2\pi f t + \underbrace{\sin(2\pi f t + \theta)}_{A \sin 2\pi f t + B \cos 2\pi f t}$ assoc triangle with hypotenuse C, angle θ, sides A, B, with $C = 1$ so $A = \cos\theta$, $B = \sin\theta$.

you need to be able to do this!

$$= \underbrace{(1 + \cos\theta)}_{\text{new } A} \sin 2\pi f t + \underbrace{\sin\theta}_{\text{new } B} \cos 2\pi f t$$

$$= \underbrace{\sqrt{(1+\cos\theta)^2 + \sin^2\theta}}_{\text{ampl.}} \sin(2\pi f t + \phi)$$

Convert back via $C = \sqrt{A^2 + B^2}$, $\phi = \tan^{-1} B/A$

so intensity $= [(1+\cos\theta)^2 + \sin^2\theta]\, I$

$$= (1 + 2\cos\theta + \underbrace{\cos^2\theta + \sin^2\theta}_{1})\, I$$

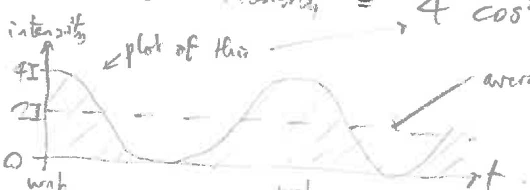
$$= 2(1 + \cos\theta)\, I$$

← note: varies from 0 to 4.

d)
2\. $g(t) = \underbrace{2\left(\cos 2\pi \frac{f_2 - f_1}{2} t\right)}_{A} \left(\sin 2\pi \frac{f_1 + f_2}{2} t\right)$ beats formula.

so intensity $= 4\cos^2\left(2\pi \frac{f_2 - f_1}{2} t\right) I$ as t increases, intensity varies from 0 to 4

e)
2\.



Average of $\cos^2$ function is $1/2$. (eg since $\int_0^\pi \cos^2\theta\, d\theta = \pi/2$)

Average intensity $= 2I$, which is the simple addition of powers $I + I$ from two musicians. So, as long as two sources of sound are not completely phase-locked (every oscillation is same for two signals, ie coherent), their powers or intensities 'add' in the naive way, when averaged over time. This is tricky!

2. [8 pts]

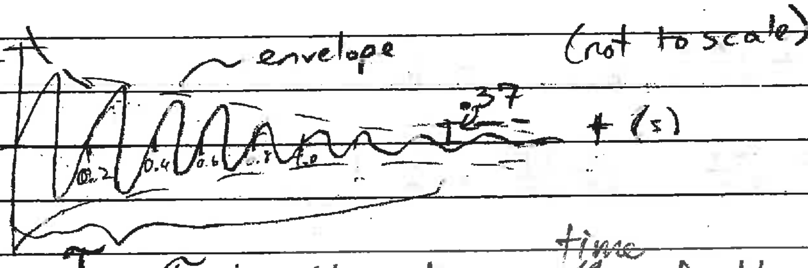
signal = $e^{-t/\tau}\sin(\omega t)$, starting $t=0$

$\tau = 2$ sec.

2. a) $\omega = \frac{2\pi}{T}$, $T = 0.2$ sec

$\omega = \frac{2\pi}{0.2} = 31.4$ rad/s

2. b)



τ is the decay ~~…~~ time of the amplitude of the signal.

c) $A = e^{-t/\tau} = e^{-4/2} = .135$

2. Amplitude at $t = 4s \Rightarrow \boxed{.135}$.

2. d) $Q = \pi \cdot f \cdot \tau = \frac{\pi}{T} \cdot \tau = \frac{\omega\tau}{2}$

$Q = \frac{\pi\tau}{T} = \frac{\pi \cdot 2}{0.2} = \frac{\pi}{1/10} = 10\pi = 31.4$

$Q = \frac{\omega \cdot \tau}{2} = \omega \cdot \frac{2}{2} = \omega$ (unitless)

(Homework #5 Solutions)

3. a) τ, decay time constant for amplitude

@ time $= \tau$ amplitude has decreased to 37% of original.

$$\frac{A_2}{A_1} = .37\cdots = e^{-1}$$

$$I \propto A^2$$

$$\frac{I_2}{I_1} = (.37)^2 = .1369$$

$$\text{dB difference} = 10\log_{10}\left(\frac{I_2}{I_1}\right) = 10\log_{10}(\underbrace{.1369}_{e^{-2}})$$

$$= -8.69 \text{ dB}$$

Intensity drops by <u>8.69 dB</u> in one decay time.

b) 10 dB difference

$$-10 \text{ dB} = 10\log_{10}\left(\frac{I_2}{I_1}\right) = 10\log_{10}\left(\frac{A_2^2}{A_1^2}\right) = 20\log_{10}\left(\frac{A_2}{A_1}\right)$$

$$-\frac{1}{2} = \log_{10}\left(\frac{A_2}{A_1}\right)$$

$$\frac{A_2}{A_1} = 10^{-1/2} = .3162$$

Takes <u>1.15 time constants</u> to drop 10 dB in intensity.

$$A_2 = A_1 \cdot e^{-t/\tau} = A_1 \cdot .3162$$

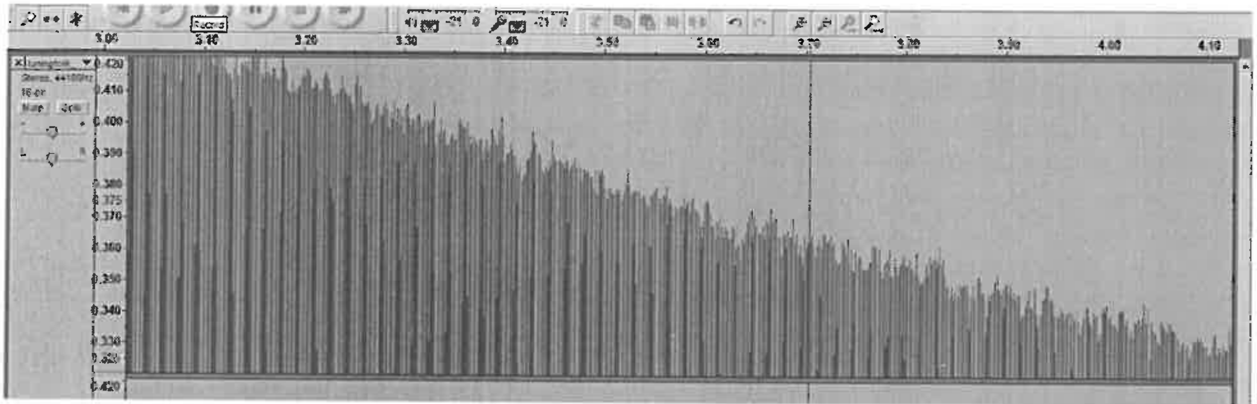
$$e^{-t/\tau} = .3162$$

$$-\frac{t}{\tau} = \ln(.3162)$$

$$t = -\tau \cdot \ln(.3162) = 1.15\,\tau$$

4

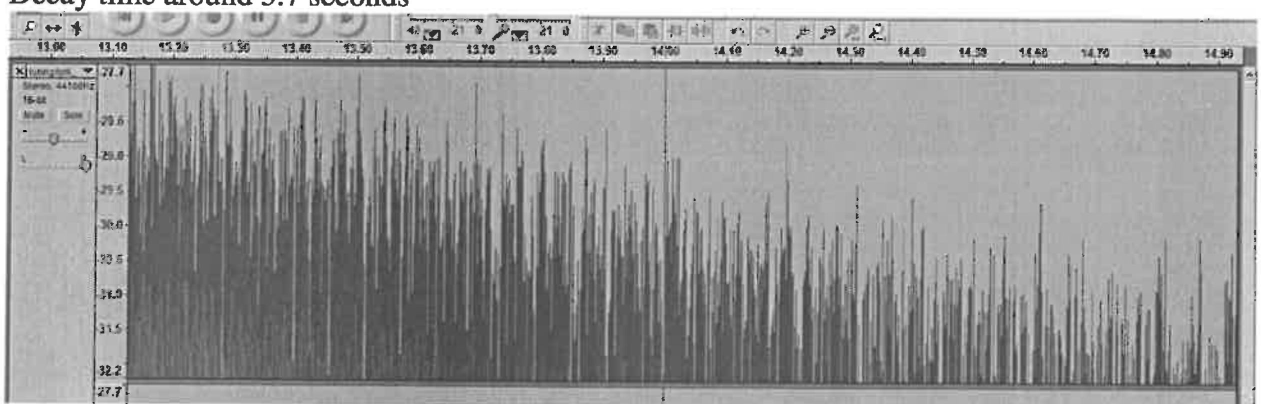
a)



A_initial = 1

A = .37 @ ~ t = 3.7 s

Decay time around 3.7 seconds



b)

A_initial = 0 dB

A = 30 dB at 14s

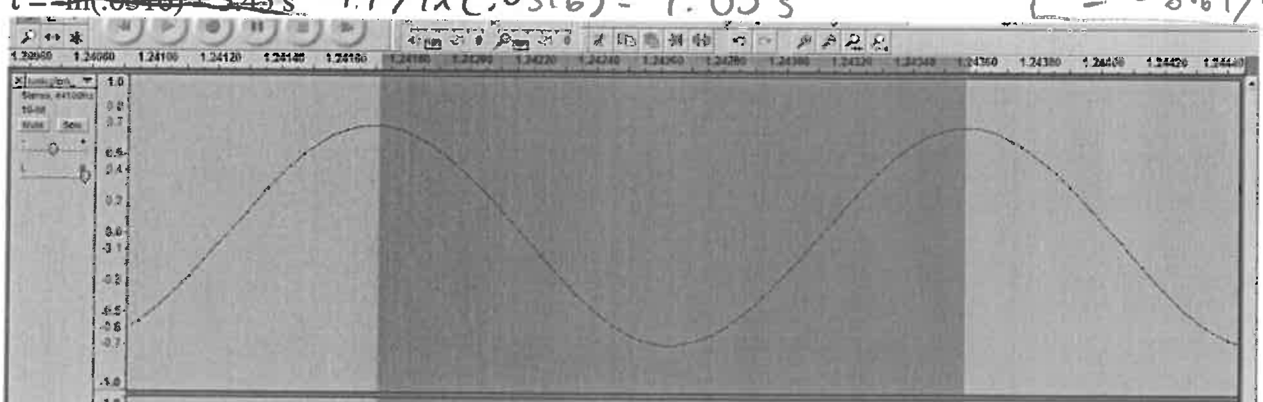
dB diff = -30 dB = 20 * log(A2/A1)

A2/A1 = 10^-1.5 = .0316 = e^(-t/τ)

τ = ~~-ln(.0316) = 3.45~~ s $-14/\ln(.0316) = 4.05$ s

note dB change btw. I_1 & I_2 is $10\log_{10}\frac{I_2}{I_1} = 20\log_{10}\frac{A_2}{A_1}$

Or: $\boxed{\text{slope in dB per unit time} = -8.69/\tau}$

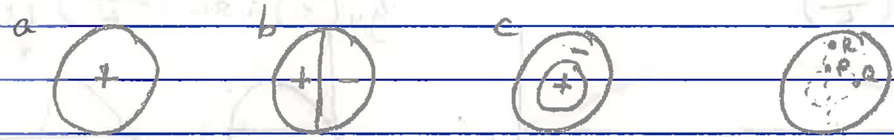


c)

τ = 3.45s, T = 1.24355 - 1.24170 = .00185 ← or use spectrum to get center freq. 534 Hz.

Q = π * τ / T = 5859

5.



P excites a (a the most)
Q excites a and b (b the most)
R excites a and c (c the most)

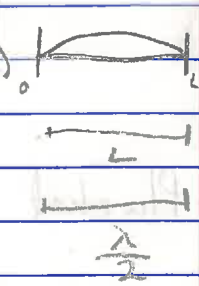
Damping at Q means only mode c may vibrate (others damped) since c has nodal line there. This is independent of where hit.

6. a) $c_{string} = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{100}{.001}} = 316\ m/s$

b) $\lambda\,(m) = c_{string}\left(\frac{m}{s}\right) \cdot \frac{1}{f}\,(s) = 316 \cdot \frac{1}{440} = .718\ m$

c) For string $\lambda_n = \frac{2L}{n}$ $\quad \lambda_1 =$ fundamental wavelength $= 2L = .718$

$y_1 = \sin\left(\frac{\pi x}{L}\right)$



$L = .359\ m$

compare 32 cm for sounding length of violin strings. Pretty close!