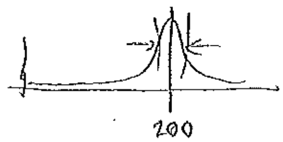
MATH 5 HW 7 2010

Barnett 11/11/10.

1 a). $f_0 = 200$ Hz



width of 50 ¢ @ 200 Hz

$\Delta f = 200 \cdot 2^{\frac{0.5}{12}}$ ← $n = 0.5$

≈ 5.8 Hz.

$Q = \frac{f_0}{\Delta f} \approx 34.5.$ $\tau = \frac{Q}{\pi f_0} = 0.056$ s. (short!)

b) $\tau = 1$ s $Q = \pi f_0 \tau \approx 628$ (large!) $\Delta f = \frac{f_0}{Q} = \frac{f_0}{\pi f_0 \tau} = \frac{1}{\pi \tau} \approx 0.318$ Hz. ← note actually f_0 irrelevant!

So, to sing & excite the wine glass, need to get freq. accurate to a fraction of a Hz. ('being maybe 5 cents), which is very hard! It's not the volume that's the problem.

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2. a&b) First mode $f_0 = 600$ Hz, $\Delta f \approx 400$ Hz $Q \approx 1.5$

Second " 1800 Hz . 200 Hz $Q \approx 9$

↳ natural freqs are ratio 1:3 so odd harmonics ⇒ closed-open.

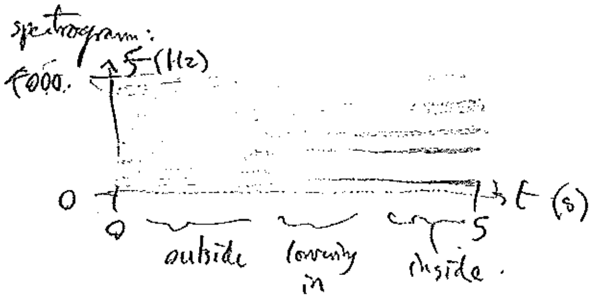
$f_n = (2n-1)\frac{c}{4L}$ use $n=2$: $f_2 = \frac{3c}{4L}$

$\Rightarrow L = \frac{3c}{4 f_2} = \frac{3(340)}{4(1800)} \approx 0.142$ m. (short vocal tract)

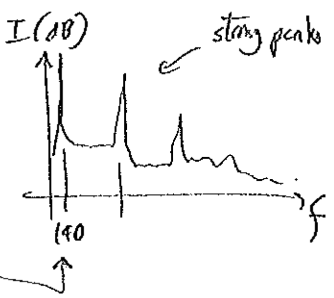
c) semis $n = 12 \frac{\log \frac{1900}{1700}}{\log 2}$ ← upper & lower $\frac{1}{\sqrt{2}}$ max amplitude points.

$= 1.9$ semitones.

3.



take spectrum of inside part:

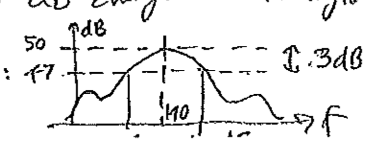


lowest-freq peak is Helmholtz resonance of bottle.

Spectrum goes from flat (uniform at all freqs.) to peaked (strong partials).

b) $A_2 = \frac{1}{\sqrt{2}} A_1$ ie $I_2 = \frac{1}{2} I_1$ so dB change $= 10 \log_{10} \frac{I_2}{I_1} = 10 \log_{10} \frac{1}{2} = -3.01$ dB

c) Zoom in on spectrum around 140 Hz :



$\Delta f \approx 2.5$ Hz

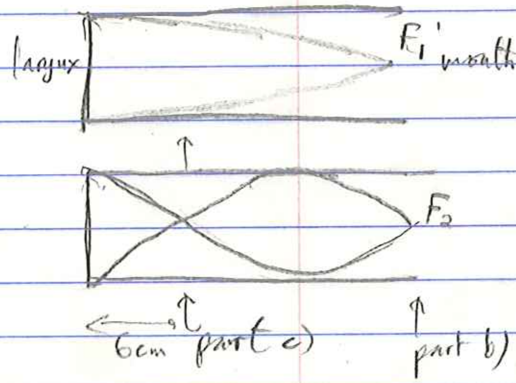
$Q = \frac{f_0}{\Delta f} \approx 56$ $\tau = \frac{Q}{\pi f_0} \approx 0.1$

14. closed-open pipe → L = .17 m

a) $f_n = \frac{n\, c_{air}}{4L}$, n = 1, 3, 5 ...

$F_1 = \frac{c_{air}}{4 \cdot L} = \frac{340}{4 \cdot .17} = 500$ Hz

$F_2 = \frac{3 c_{air}}{4 \cdot L} = 3 \cdot F_1 = 1500$ Hz



b) If mouth is opened wider, how does it affect F_1 and F_2? Mouth is the open end, with a node for F_1 and F_2.

Rule:	Wider @ node	frequency ↑
	Narrower @ node	frequency ↓
	Wider @ anti-node	frequency ↓
	Narrower @ anti-node	frequency ↑

So F_1 and F_2 will rise in frequency.

c) Pharynx constricted @ x = 6cm → .35 of the way from vocal cords

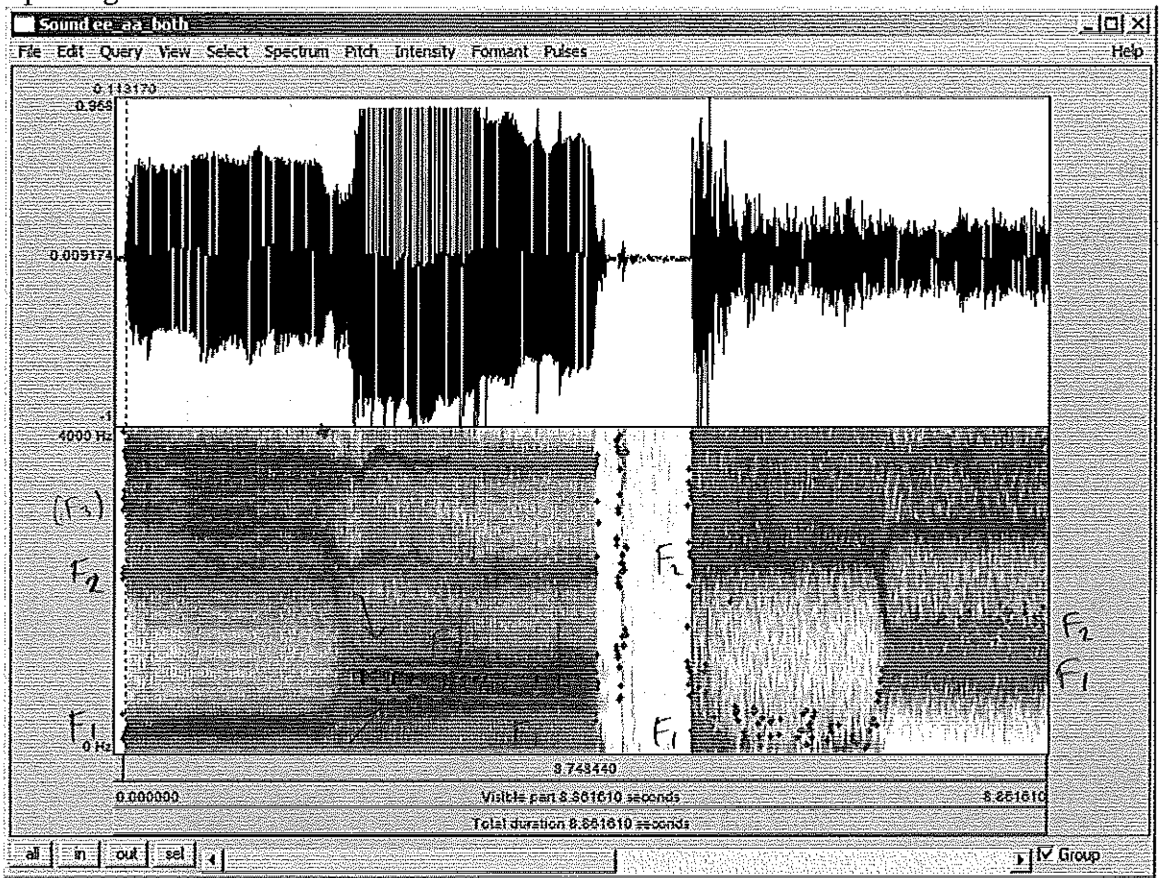
As noted, F_1 has an anti-node at x = 0, so the pharynx constriction is much closer to the anti-node than the node @ x = 17cm. F_2 has a node at x = 5.6cm, so the point x = 6 is approximately at a node = constriction @ node = frequency down.

So, F_1 will move up, F_2 will move down.

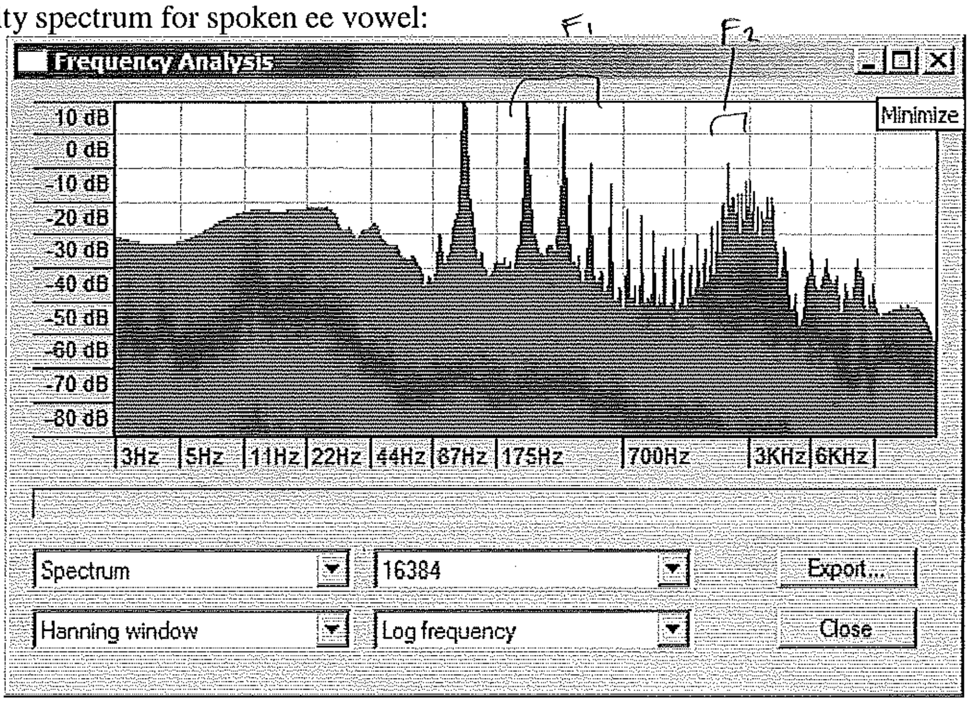
6.

5.

Spectrogram with formants shown:



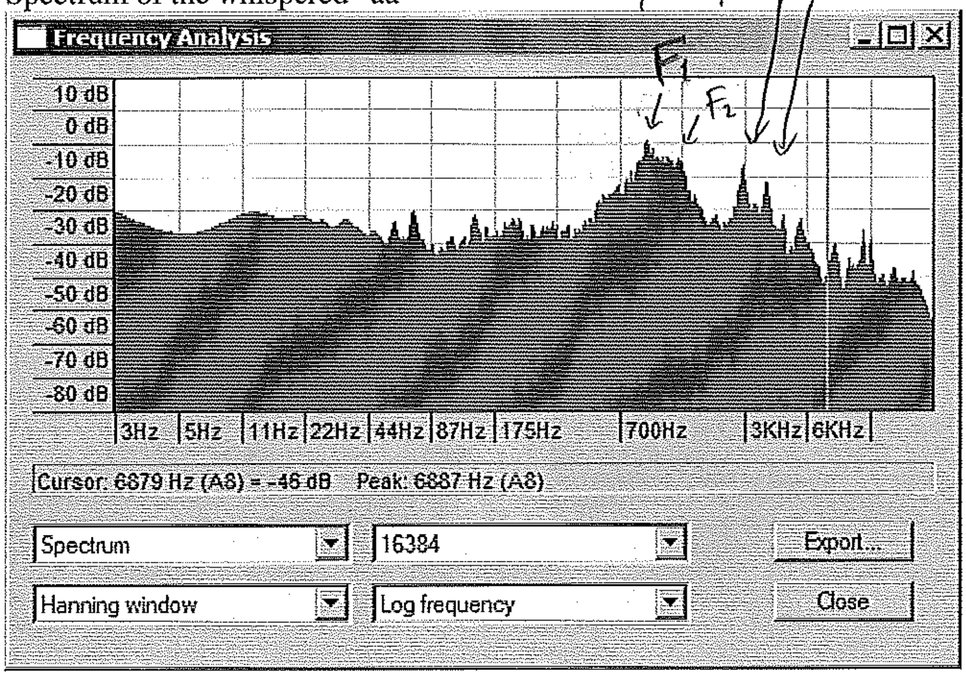
Audacity spectrum for spoken ee vowel:



a)
"EE" -- Fundamental frequency (from Audacity) = 123 Hz
F1 = 350 Hz F2 = 2250 Hz

"AA" -- Fundamental frequency = 122 Hz
F1 = 727 Hz F2 = 1035 Hz

b)
Spectrum of the whispered "aa"

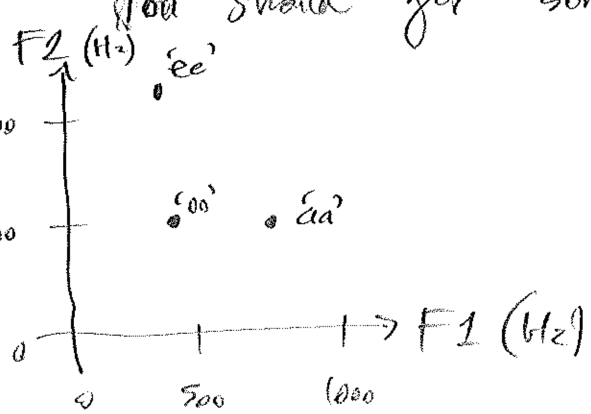


No apparent pitch, very even spread. { ie no v. narrow peaks, just colored white noise. No comb of equally-spaced partials at f, 2f, 3f, ... which is needed for perceived pitch.

Although there is not a pitch, the spectrogram picture shows strong upper formant lines. The formants are a similar shape and close to the same frequency whether whispered or spoken. → (although F1 for 'ee' is much weaker since whispering doesn't excite low freqs. v. much. F1 & F2 for 'aa' move up a bit).

#6. a) similar to mine (with 'oo' having lower formants than 'aa').

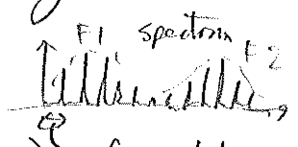
b) You should get something like:



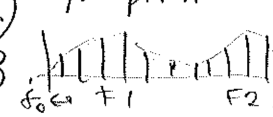
(varies by about 25%, dep. on gender)!

[See Johnston book on reserve, or Rossing book]

Note some of you mistook F3 (≈2300) for F2 in 'oo', which has both F1 & F2 low.



c) pitch will not change (spacing of harmonics) But formants F1 & F2 will be multiplied by eg. 1.3



alternative: n unknown, want $\frac{n+1}{n} = 2^{1/12}$ for 1 semitone.

so $\frac{1}{n} = 2^{1/12} - 1 \quad n = (2^{1/12} - 1)^{-1} = 16.8.$

So $n \approx 17$ is good.

7. a) How high up so that adjacent harmonics are separated by whole tone + semitone?

List of harmonics

Also you know Pythag. whole tone is 9/8.

f, 2f 3f 4f 5f 6f 7f 8f 9f 10f 11f

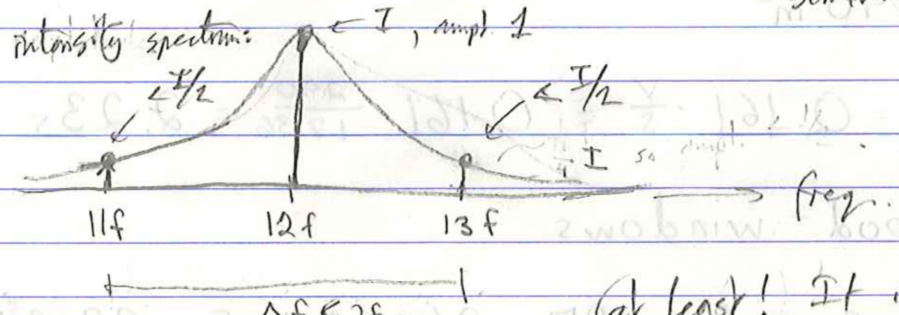
Frequency ratios:	1	2	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{5}{4}$	$\frac{6}{5}$	$\frac{7}{6}$	$\frac{8}{7}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{11}{10}$	$\frac{12}{11}$	$\frac{13}{12}$	$\frac{14}{13}$
Semitones:							2.66	2.3	(2.04)	1.82	1.65	1.51	1.38	1.28

→

$\frac{15}{14}$	$\frac{16}{15}$	$\frac{17}{16}$	$\frac{18}{17}$
1.19	1.12	1.05	(.99)

✗ Note: Tuvans usually sing up to 12th harmonic, somewhere between the above 9th & 18th.

b) intensity spectrum:

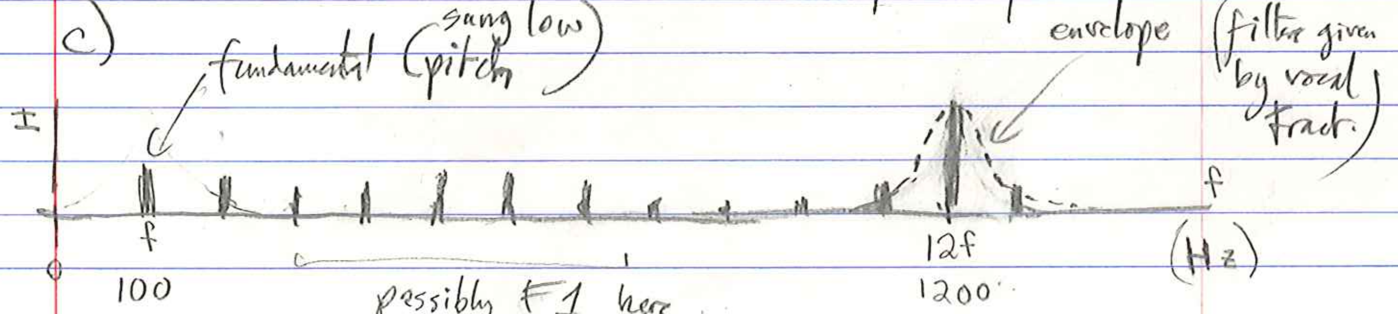


(at least! It is often much higher)

$$Q > \frac{12f}{2f} = 6$$

c)

narrow formant due to strange shape of singer's vocal tract.



7. room : $4 \times 6 \times 10\ m^3$

a) $T_{60} = 0.161 \cdot \frac{V}{S}$ $\quad V = 240\ m^3$

$A = 2(4\times6) + 2(4\times10) + 2(6\times10)\ m^2$

$= 248\ m^2$

$S = \sum_i \alpha_i A_i$

Perfect reflection ⇒ $\alpha = 0$ $\quad 243\ m^2$

Open window ⇒ $\alpha = 1$ $\quad 5\ m^2$

$S = 0 \cdot 243 + 1 \cdot 5 = 5$

$T_{60} = 0.161 \cdot \frac{240}{5} = 7.728\,s$

b) Whole room is wood, no windows, $\alpha = .07$ @ 1000 Hz

$S = \sum_i \alpha_i A_i = .07 \cdot 248 = 17.36$

$V = 240\ m^3$

$T_{60} = 0.161 \cdot \frac{V}{S} = 0.161 \cdot \frac{240}{17.36} = 2.23\,s$

c) wood + windows

$S = \sum_i \alpha_i A_i = .07 \cdot 243 + 1 \cdot 5 = 22.01\ m^2$

$T_{60} = 0.161 \cdot V/S = 0.161 \cdot \frac{240}{22.01} = 1.77\,s$