Math 5: Music and Sound. Homework 8

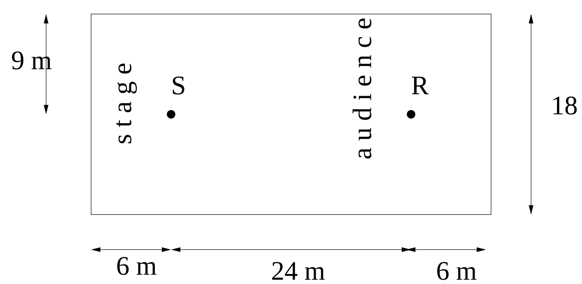
due Wed Nov 16

Half the usual length to give you time to work on projects!

1. Measure the reverberation time T_{60} of the Kemeny stairwell from the sound of a clap: `Kemeny_stairwell_clap.wav` I suggest you use the `Show intensity` feature of `praat` and compute it from the time to drop by only 30 dB. Quote your answer with an estimate on accuracy, *e.g.* 5 ± 0.7 sec.

2. Here's a musically important example of diffraction of sound. A trombone's bell (where the majority of sounds radiates from) is about 17 cm in diameter.
   (a) Which frequency range do you expect to radiate roughly in all directions from the bell?
   (b) Which frequency range do you expect to emerge like a directed beam from the bell?
   (c) The trombone plays a 400 Hz pitch with lots of high harmonic strength. Compute the angular width of the beam for the 20th harmonic of this note.
   (d) Use this to comment on the expected difference in timbre for listeners standing along the axis of the trombone compared to those off to the side.

3. Here's a simple auditorium shown in plan view, with a performer (source S) and audience member (receiver R):



   (a) Our ears meld together echoes separated by less than about 0.05 s. Find the time difference in ms between the arrival of direct sound and the *first* reflection off the walls. [Show this on a diagram. Only consider reflections in the plane shown.]
   (b) What is the lowest pure tone frequency that would cause *destructive* interference between the direct and first reflected paths? BONUS: what is the formula for all such frequencies?
   (c) Use the method of images to compute the arrival times and draw the paths corresponding to the second, third and fourth echo arrival times.

4. Here you explore how *nonlinearity* affects sound reproduction. Sketch the pure tone signal $y = f(t) = \sin(200\pi t)$ for $0 < t < 0.02$. We will transform the signal by composing with another function g or h. This represents distortion by electronics such as a tube amplifier (which by the way is claimed to be beneficial by the tube faithful), or bad loudspeaker.
   (a) Let $g(y) = 2y$. Is $g(y)$ a linear or nonlinear function? Write the composed function $g(f(t))$ as a function of t alone (by substituting in for y). Add $g(f(t))$ to your sketch. Has the signal shape changed?

(b) Now let $h(y) = y + y^2$. Is $h(y)$ a linear or nonlinear function? Write $h(f(t))$ as a function of t alone, and add it to your sketch. (You may want to use `fooplot.com` or some such to get the sketch right). Has the shape changed? Since $h(f(t))$ is also periodic (what is its period?) it can be written as a Fourier series. Use a trigonometric identity (*e.g.* look in my math review notes) to reduce it to a sum of pure sinusoids, and give the resulting c_j amplitudes. Are any frequencies present that were *not* in the original signal? How could this change the timbre?