MATH 5 : All you need to know about the Doppler effect

4/18/07

Key results :

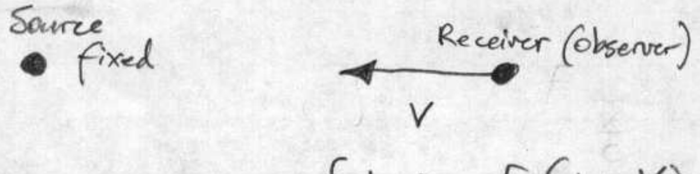
i)

c = speed of sound.

$$f_{obs} = \frac{f}{1 - v/c}$$ source towards you

$$f_{obs} = \frac{f}{1 + v/c}$$ source away from you.

ii)



$$f_{obs} = f\left(1 + \frac{v}{c}\right)$$ receiver towards source

$$f_{obs} = f\left(1 - \frac{v}{c}\right)$$ receiver away from source.

Notes : In each case the 'away from' formula is gotten by setting $v \to -v$ in the 'towards'. So there's only actually 2 formulae to know, not 4.

'Fixed' and 'moving' are relative to the air (if no wind, also the ground).

In case i) above, you get a sonic boom if $v = c$ since $f_{obs} = \frac{f}{0} \to \infty$!

That's it. You don't need the two-dimensional Doppler in the book (it's wrong anyway).

You don't have to know the derivation of the above.

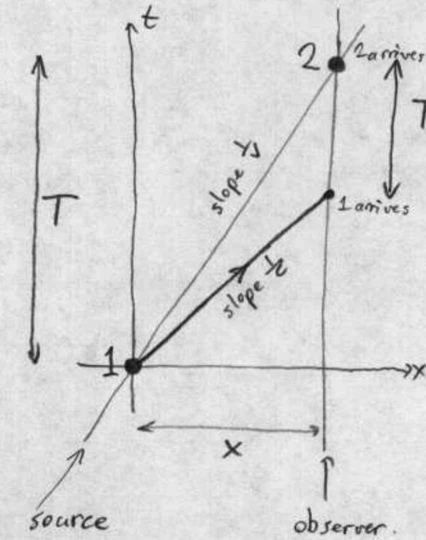
You might have to solve for v given f/f_{obs} and c.

Eg $f = 450$ Hz , $v_{source} = 34$ m/s towards so $f_{obs} = \frac{450}{1 - \frac{v}{c}} = \frac{450}{1 - 0.1} = 500$ Hz, you hear

but if source fixed, $v_{observer} = 34$ m/s towards , $f_{obs} = f(1 + \frac{v}{c}) = 450(1.1) = 495$ Hz you hear.

Now to clear any confusion let's derive them (as in class) ...

i) Moving source, fixed observer. (essence of worksheet from class)



Pulse 1 launched at $t=0$ from $x=0$,
arrives at location x at time $t=\frac{x}{c}$

Pulse 2 launched at $t=T$ from location x.
it is already at the observer so instantly arrives at $t=T$

Period between arrivals $T_{obs} = \overbrace{T}^{\text{pulse 2}} - \overbrace{\frac{x}{c}}^{\text{pulse 1}}$

But must have $x = vT$ by diagram.

$$\Rightarrow \quad T_{obs} = T - \frac{vT}{c} = T\left(1-\frac{v}{c}\right)$$

reciprocal & divide by f.

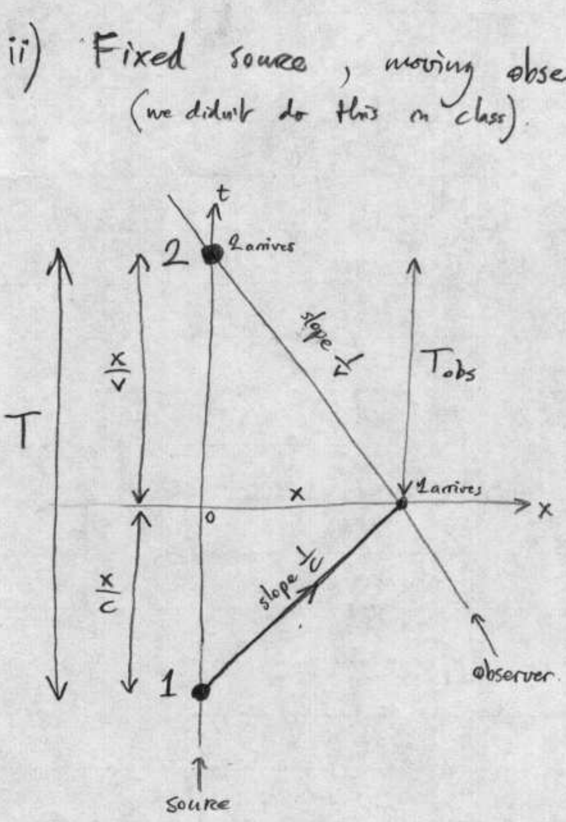
$$\boxed{\frac{f_{obs}}{f} = \frac{1}{1-v/c}}$$

Doppler frequency change

using standard result $f=\frac{1}{T}$ for periodic signals.

this works for $v>0$ (towards) or $v<0$ (away).

ii) Fixed source, moving observer
(we didn't do this in class).



Pulse 1 launched at $t=-\frac{x}{c}$ at location 0
arrives at location x at time $t=0$

Pulse 2 launched at $t=+\frac{x}{v}$ at location 0
instantly arrives since observer also there.

So $T = \frac{x}{v}+\frac{x}{c}$ but $T_{obs} = \frac{x}{v}$

$$\frac{f_{obs}}{f} = \frac{T}{T_{obs}} = \frac{\frac{x}{v}+\frac{x}{c}}{\frac{x}{v}} = 1+\frac{v}{c}$$

so

$$\boxed{\frac{f_{obs}}{f} = 1+\frac{v}{c}}$$

Doppler for moving receiver.

as before, $v>0$ (towards)
$v<0$ (away)