MATH 5 WORKSHEET : Groups and melodies

12/1/08
Bennett.

- Complete the group table giving the operation equivalent to each combined pair :

first do / then do	E	I	T	C
E		I		
I			C	
T				
C				

E = identity
I = inversion $(y \to -y)$
T = reversal $(x \to -x)$
C = 'crab' $(x \to -x, y \to -y)$ ie π rotation.

- A group is called 'Abelian' if it doesn't matter which order any two operations are performed in. Is this group Abelian?
What symmetry of the above 4x4 matrix reflects* this? [*no pun intended...?]

- Consider another group of the 4 operations below acting on the numbers $\{0,1,2,3\}$:
  - a : subtract the number from 3
  - b : add 2 (& if >3 then subtract 4)
  - d : do nothing
  - f : if even add 1 ; if odd subtract 1.

Each of the operations a, b, d, f is equivalent to one of E, I, T, C ... Which is which? [Hint: a acting on $\{0,1,2,3\}$ gives $\{3,2,1,0\}$. What does ab give? Make the table]

- The above group was 'isomorphic' to the EITC group. (can show they have the same table).
How about
  - a : add 1 (& if >3 subtract 4)
  - b : add 2 (& if >3 subtract 4)
  - d : add 3 (& if >3 subtract 4)
  - f : do nothing

[Hint : make the table].

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12/1/02. Bamett.

SOLUTIONS

- Complete the group table giving the operation equivalent to each combined pair :

first do / then do	E	I	T	C
E	E	I	T	C
I	I	E	C	T
T	T	C	E	I
C	C	T	I	E

E = identity
I = inversion ($y \to -y$) (swap low for high pitches)
T = reversal ($x \to -x$) (retrograde, ie time reversal)
C = 'crab' ($x \to -x$, $y \to -y$) ie π rotation.

reflects along diagonal ⇒ order doesn't matter.

- A group is called 'Abelian' if it doesn't matter which order any two operations are performed in. Is this group Abelian? Yes (eg CT=TC)

What symmetry of the above 4×4 matrix reflects* this?

diagonal reflection.

[*no pun intended...?]

- Consider another group of the 4 operations below acting on the numbers {0,1,2,3}:

a : subtract the number from 3 {0123} → {3210}
b : add 2 (& if >3 then subtract 4) {0123} → {2301}
d : do nothing {0123} → {0123}
f : if even add 1 ; if odd subtract 1. {0123} → {1032}

Each of the operations a,b,d,f is equivalent to one of E,I,T,C ... Which is which?

(Hint: a acting on {0,1,2,3} gives {3,2,1,0}. What does ab give? Make the table]

Table

	a	b	d	f
a	d	f	a	b
b	f	d	b	a
d	a	b	d	f
f	b	a	f	d

d = E (identity)
a = C
b = T
f = I

(any!) although other choice also works here, this one corresponds to binary ops on {00, 01, 10, 11}!

ab → {1032} = f.

- The above group was 'isomorphic' to the EITC group. (can show they have the same table).

How about
a : add 1 (& if >3 subtract 4)
b : add 2 (& if >3 subtract 4)
d : add 3 (& if >3 subtract 4)
f : do nothing

	a	b	d	f
a	b	d	f	a
b	d	f	a	b
d	f	a	b	d
f	a	b	d	f

[Hint: make the table].

cannot be mapped onto above table → not isomorphic.