① 10/21/10

~ SOLUTIONS ~

Math 5: Music and Sound, 2010. Midterm

2 hours, 6 questions, 60 points total

Please show working. Points per question is shown to help judge time. Useful info on last page. Good luck!

1. [9 points + bonus]

2 (a) Find the frequency of D7 in the equal-tempered system.

[2 octaves above D5 which is itself 5 semis above A4. Total = 2(12) + 5 = 29 semis.

$f_{D7} = (440)2^{\frac{29}{12}} \approx 2349$ Hz

3 (b) When you press "1" on a land-line phone you hear two pure tones at 1209 Hz and 697 Hz. Convert this into a musical interval in semitones and state between which Western intervals it lies.

interval

$n = 12 \dfrac{\log \frac{1209}{697}}{\log 2} = 9.54$ semitones

note = ratio.

this is between major 6th & minor 7th.

4 (c) Jeff Zeigler, the cellist of the Kronos Quartet, explained that if he tunes the top A string of his cello to A3 (220Hz), then the D string a Pythagorean perfect fifth below that, the G string the same fifth below that, and the C string the same fifth below that, then this last string ends up being out of tune with an equal-tempered piano playing the same C note (e.g. when playing chamber music together). Compute how far, in cents, and is the cello sharp or flat?

$C2 \xleftarrow{3/2} G2 \xleftarrow{3/2} D3 \xleftarrow{3/2} A3$

going down three times (21 semitones)

$f_{C2,\text{Zeigler}} = (220)\left(\frac{2}{3}\right)^3 = 65.185$ Hz

One way to convert its difference in cents from equal temperament is to convert this to equal-tempered cents:

$\text{cents from A3} = 1200 \dfrac{\log \frac{f_{C2,\text{Zeigler}}}{220}}{\log 2} = -2105.9$ cents.

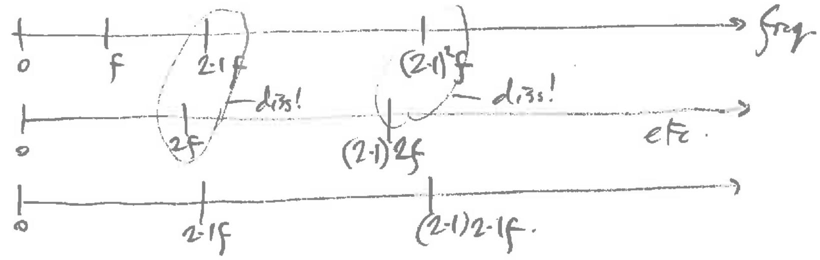
cello is 5.9 cents flat.

BONUS: Explain one way in which the ear can be persuaded that an octave sounds *more* out of tune than some other nearby non-octave frequency ratio.

'Stretching' the partials :

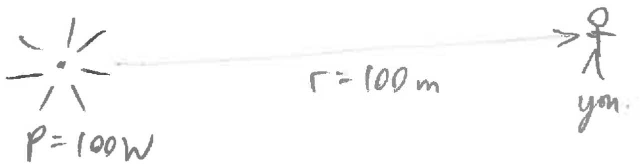
is dissonant against 2f :

but consonant w/ 2·1f :



2. [13 points + bonus]

3 (a) At an outdoor concert, a single loudspeaker puts out 100 W of acoustic power with a pure tone of 170 Hz (beautiful music!) You stand a distance 100 m from this speaker. How many decibels are at your location? (assume equal radiation in all directions.)



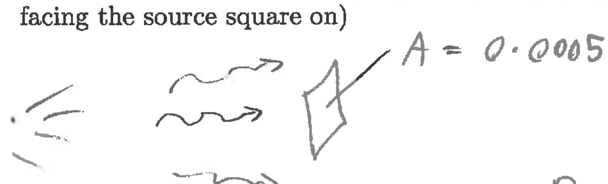
$$I = \frac{P}{4\pi r^2}$$

$$= \frac{100}{4\pi\, 100^2} = 7.96\times10^{-4}\ \mathrm{W/m^2}$$

Then,

$$dB = 10\log_{10}\frac{I}{I_r} = 10\log_{10}\frac{7.96\times10^{-4}}{10^{-12}} = 89.0\ dB$$

2 (b) How much power is thus impinging on the area of your ear? (take this area as 0.0005 m^2, and facing the source square on)



$$I = \frac{\text{Power}}{\text{Area}}$$

so $\text{Power} = I\cdot\text{Area} = (7.96\times10^{-4})\,0.0005$

$$\approx 3.98\times10^{-7}\ \mathrm{W}\ (\text{small!})$$

3 (c) You move closer to a distance of 50 m. How do the decibels change relative to part (a)? [Hint: you can answer this without having answered part (a)]

Use ratios : with P held constant: $I_1 = \frac{P}{4\pi r_1^2}$, $I_2 = \frac{P}{4\pi r_2^2}$

so $\frac{I_2}{I_1} = \frac{\cancel{4\pi} r_1^2}{\cancel{4\pi} r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{100}{50}\right)^2 = 2^2 = 4.$

dB increase $= 10\log_{10}\left(\text{intensity ratio}\right) = 10\log_{10}4 = \underline{6.02\,dB}$

(d) A second loudspeaker is added a distance 1 m behind the first (i.e. further from you), driven with exactly the same 170 Hz signal. Use travel times to compute the phase difference between the pure tones arriving at your location from each speaker.

3

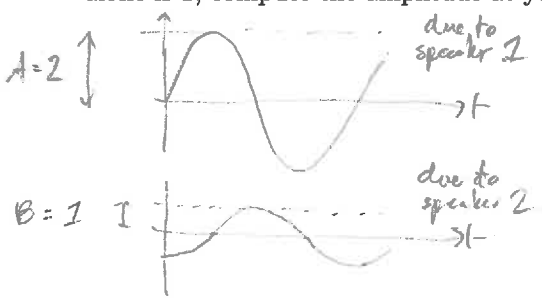
$$\phi = -\frac{\omega x}{c} = -\frac{2\pi f x}{c} = -\frac{2\pi(170)1}{340}.$$

$= -\pi$ rad, or $+\pi$ (since doesn't matter)

(exactly out of phase).

(e) A pure tone at a new frequency is sounded at which the phase difference in part (d) becomes $\pi/2$. If the amplitude at your location due to the first speaker alone is 2, and due to the other speaker alone is 1, compute the amplitude at your location when both speakers are on.

2



Phase difference is $\pi/2$ so we may treat one as sin & the other as cos.

→ want ampl. of $\underset{A}{2}\sin\omega t + \underset{B}{1}\cos\omega t$.

$C = \sqrt{A^2 + B^2} = \sqrt{5}$ new amplitude.

BONUS: Say the amplitude from these two speakers is adjusted to be equal where you are standing. How do you think the effect you deduced in part (d) will change the *spectrum* of any music played through this system? Sorry!

Effect in part (d) is cancellation of any components at $f = 170$ Hz, 510 Hz, 850 Hz, ...
$(2n+1)170$ Hz, $n = 0, 1, 2, \ldots$

Effect in part (e) just makes everything louder, so spectrum will be same, but uniformly louder.

3. [11 points] A police car siren produces a note (you may assume pure tone) with frequency 500 Hz.

(a) You are stationary. How fast, and in which direction (towards or away) does the car have to drive for you to hear a frequency of 400 Hz?

3.

lower, so car goes away.

$$f_{obs} = \frac{f}{1 - \frac{v}{c}} \quad \text{ie} \quad 400 = \frac{500}{1 - \frac{v}{c}} \quad \overset{\text{solve for } v}{\Rightarrow} \quad 1 - \frac{v}{c} = \frac{500}{400} = \frac{5}{4}.$$

$$\Rightarrow \frac{v}{c} = 1 - 5/4 = -1/4, \quad v = -c/4 = -85\,\text{m/s}.$$

indicates moving away.

2

(b) Describe the only situation (moving car or moving observer) and the relevant speed for which this siren could be observed as a *zero* frequency signal? (In reality your ear won't hear it below 20 Hz, as you know; ignore this)

moving src : $f_{obs} = \dfrac{f}{1-\frac{v}{c}}$

moving listener : $f_{obs} = f\left(1+\frac{v}{c}\right)$

only this can give $f_{obs} = 0$, and it needs $v = -c$.

Car stationary, observer moving away at 340 m/s.

3

(c) A typical person cannot distinguish pitch differences of less than 10 cents. What is the minimum speed the car needs travel at to create this pitch increase relative to a siren at rest?

10 cents (0.1 semitones) corresponds to freq ratio $2^{\frac{10}{1200}} = 1.005793\ldots$

This must equal $\dfrac{f_{obs}}{f} = \dfrac{1}{1-\frac{v}{c}}$ so $\dfrac{v}{c} = 1 - \dfrac{1}{1.005793\ldots}$

$= 0.005759$

so $v = 1.96$ m/s (jogging speed)

3

(d) Given the situation of part (c) (observer hearing pitch of siren on a moving car 10 cents above 500 Hz), another stationary police car also puts on their siren at 500 Hz. What will you hear? (Give all new resulting frequencies of phenomena perceived)

Two pure-tone signals combined : 500 Hz (stationary car) $\leftarrow f_1$

& $\underbrace{500\left(2^{\frac{10}{1200}}\right)}_{= 502.9\text{ Hz}}$ (moving car) $\leftarrow f_2$

They are within ~15 Hz of each other.

→ You will hear beating at around 2.9 Hz, $= |f_2 - f_1|$

and an average pitch of $\dfrac{500 + 502.9}{2} = 501.4$ Hz.

needs both avg. freq. (tone heard) & beat ("wah" amplitude modulation) freq.

4. [7 points + bonus]

(a) Draw a space-time diagram showing why a flutter echo is heard by a listener standing exactly *half way between* two walls separated by distance L, when they produce a short sound such as a clap. Label the axes, the walls, and any sound pulse(s):

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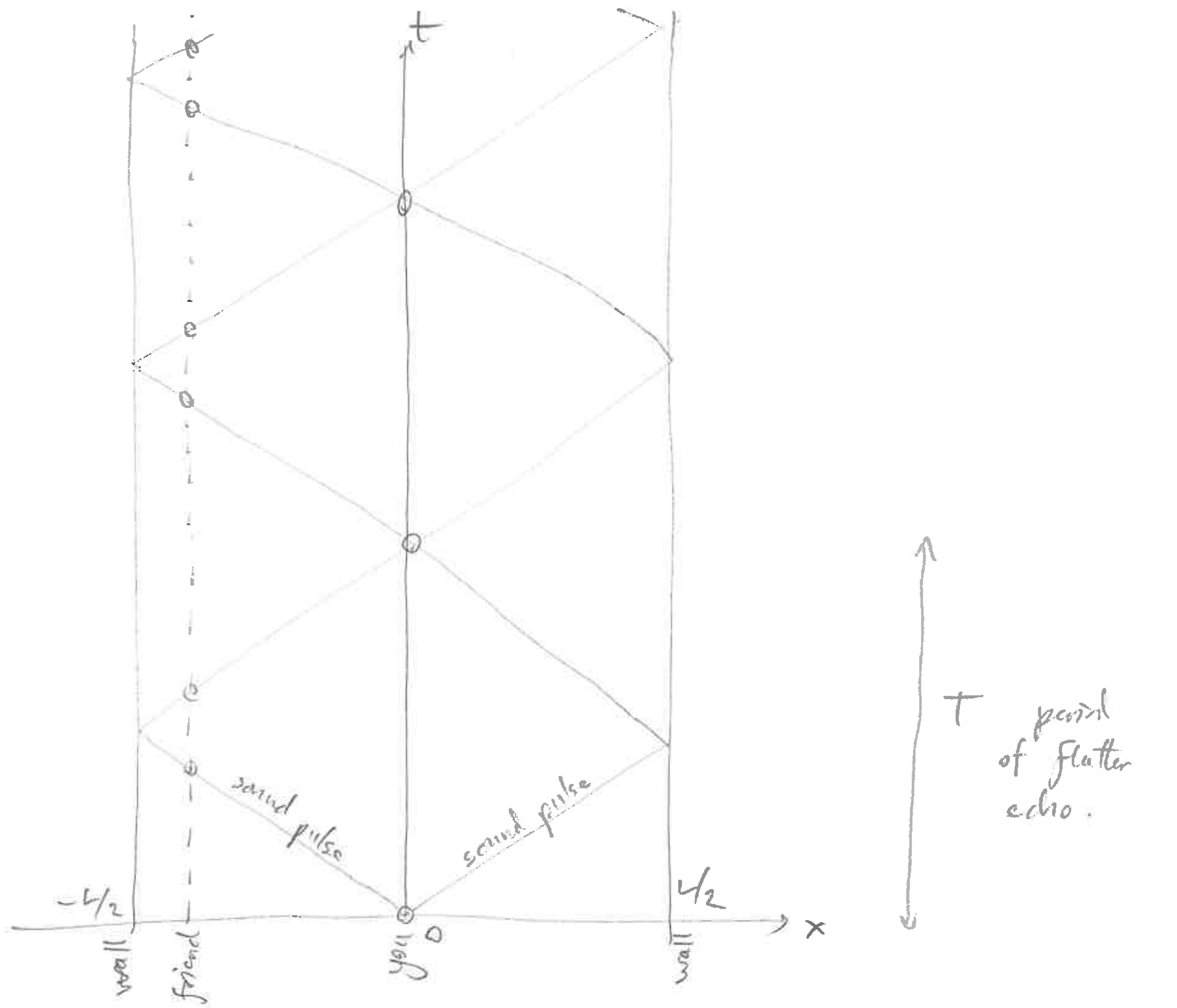


(b) What period of signal is heard if the spacing between the walls is $L = 34$ meters? [Hint: argue using your diagram, and take care]

2

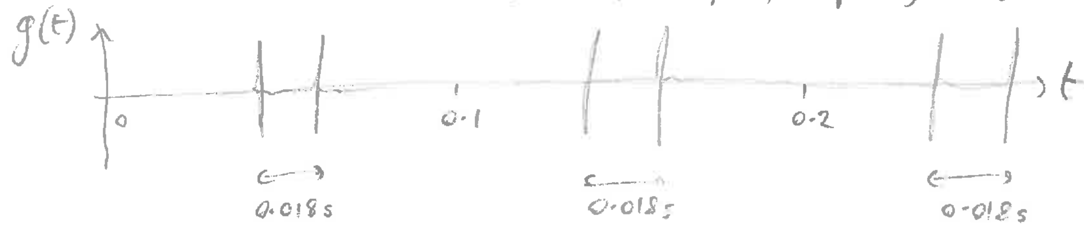
sound pulse travels $\frac{L}{2}$ twice per period T

so $T = \frac{L}{c} = \frac{34}{340} = 0.1$ sec.

(If you weren't exactly in middle, period = 0.2 sec, but this was stated).

BONUS: A friend stands 3 meters from one of the walls and listens. Sketch the signal that they would hear due to the *original* person's clap.

pairs of claps, repeating every 0.1 sec:



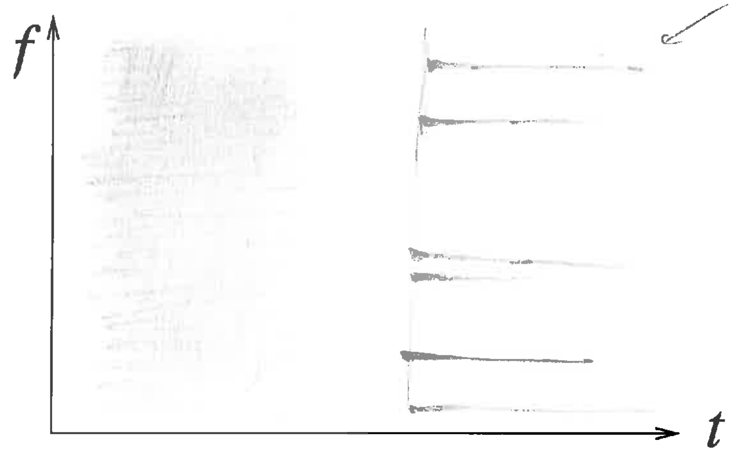
(read from 'friend' line in spacetime diag.)

5. [9 points] Sketch spectrograms on the axes provided which could realistically match the following descriptions. Feel free to explain any features in words too:

4

(a) A hissing sound, followed by struck bell of no definite pitch.

decay over time.



all freqs represented (no partials)

partials, no harmonic relationships (not f, 2f, 3f...).

4

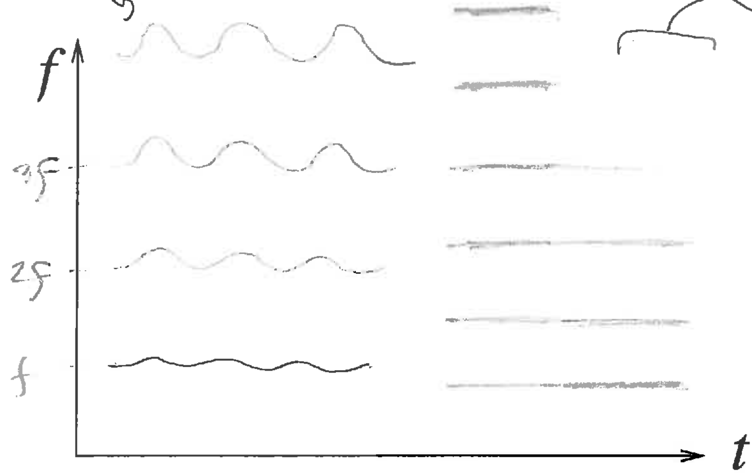
(b) A string instrument playing a single note with heavy vibrato (time-dependent variation in frequency), followed by a voice singing a single note while changing from a harsh to a mellow timbre (without changing amplitude)

slow oscillation in frequency

(Kronos Quartet: remember viola solo).

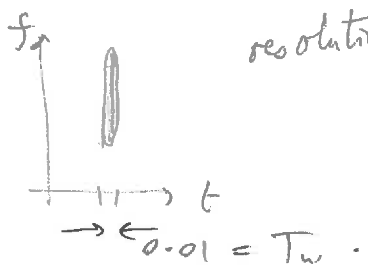
harsh (strong high harmonics)

mellow (strong low harmonics) little or no high harmonics.



1

(c) Say you wanted to use a spectrogram to capture events rapidly changing in time, thus choose a short time window of 0.01 sec, then what (if any) restrictions does this place on the accuracy with which you could resolve (distinguish) frequencies?



resolution $\Delta f \approx \frac{1}{T_w} = \frac{1}{0.01} \approx 100\,Hz$

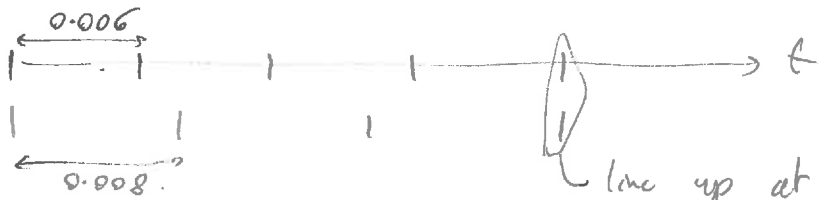
so you would get very poor frequency resolution.

6. [11 points; note 4 are for part (a) alone] Periodic and other signals.

4 (a) Place a check mark beside whichever of the following are true (could be all, some, or none): true

- (×) Any periodic signal of period $1/f$ can be written $a_1 \cos 2\pi f + a_2 \cos 4\pi f + \ldots$ ← only for even periodic signals.
- ✓ Any periodic signal of period $1/f$ can be written $a_1 \sin(2\pi f + \phi_1) + a_2 \sin(4\pi f + \phi_2) + \ldots$ Fourier series.
- (×) A signal g for which $g(t+T) = g(t)$ for all t must have period T ? no: could be $T/2$, $T/3$, etc. See defn. of Period.
- ✓ Any signal containing only partials at f, $2f$, $3f$, ... must be periodic with period $1/f$ True

2 (b) If one signal with period 0.006 s is played on top of (added to) another signal with period 0.008 s, what is the period of the resulting signal?



$= 0.024$ s.

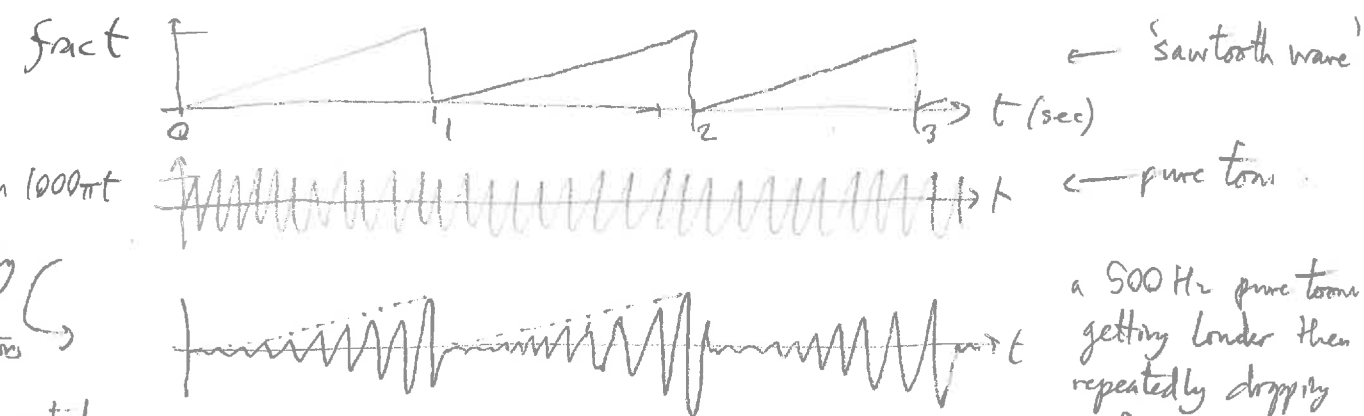
Every 0.024 s they line up again, so this is combined period.

3 (c) Explain the difference between timbre and amplitude. (Be as precise as possible; you may, and probably should, refer to other concepts from the course.)

Amplitude: size of (pressure) oscillations in a signal, eg. $A \sin \omega t$, A is amplitude. Controls how loud a sound is. Increasing ampl. would increase $c_1, c_2, \ldots$ equally.

Timbre: relative strengths of harmonic content $c_1, c_2, \ldots$ controls quality (harsh / mellow) of sound, independent of its overall amplitude or pitch. Enables different instruments to be distinguished (or, helps).

2 (d) Describe roughly what the signal $g(t) = (\text{frac}\, t) \sin 1000\pi t$ will sound like (t is time in seconds), where frac t means the fractional part of t. [Hint: a sketch may help, and could earn credit]



a 500 Hz pure tone getting louder then repeatedly dropping back to zero volume, once per second.

(frac t controls amplitude, or envelope, of the pure tone).