Barnett 10/6/11.

Solutions

Math 5: Music and Sound. Quiz 1 (2011)

30 mins (4 questions. Question 4 is worth more than Question 3)

Please write on this paper, show your working. The last page has useful information.

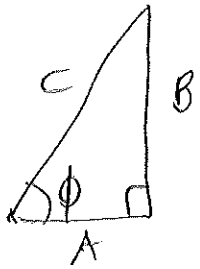
1. Consider the signal $3\sin(100\pi t + \pi/4)$ — phase. ampl. $2\pi f$ 100π must be $2\pi f$ for a sinusoid (pure tone).

   (a) What is its period?

$100\pi = 2\pi f$

$\Rightarrow f = \frac{100\pi}{2\pi} = 50$ Hz $\Rightarrow T = \frac{1}{f} = 0.02$ s.

   (b) Rewrite the signal $3\sin(100\pi t) + 4\cos(100\pi t)$ in the form $C\sin(\omega t + \phi)$. (You can leave C and ϕ as expressions). A B. ← 2 pure tones at same freq.



$C = \sqrt{A^2 + B^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5.$

$\tan\phi = B/A = 4/3$ so $\phi = \tan^{-1} 4/3$

$= 53°$

or 0.927 rad

2. (a) What musical pitch (give name and octave, *e.g.* D#3) is nearest the frequency 1109 Hz?

$n = 12 \dfrac{\log \frac{1109}{440}}{\log 2} = 16.0042\ldots$ very close to $n = 16$.

16 semis above A4 = 4 semis above A5

= C#6 (by counting on kbd)

(b) Compare the Pythagorean whole tone (9:8) and the equal-tempered whole tone, expressing their difference in *cents*.

Neatest way is to know that equal-tempered whole tone has $n=2$ so is exactly 200 cents.

How many cents is $9/8$? $n_{cents} = 1200 \dfrac{\log 9/8}{\log 2} = 203.91\ldots$

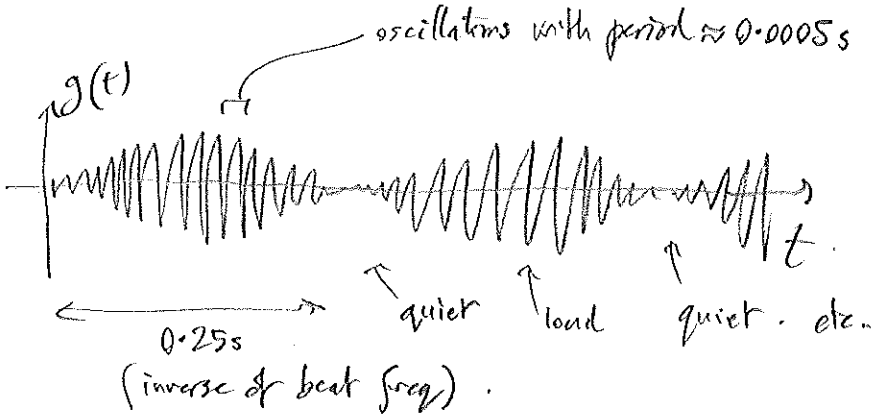
Subtracting 200 cents gives 3.91 cents (Pythag. is sharp of equal temp.)

3. What would you hear if two pure tones at frequencies 2000 Hz and 2004 Hz but the same amplitude were played together? (For full points you must state all relevant new frequencies of phenomena which occur. But you do not need to write out any trig formulae.)

$f_1 = 2000$, $f_2 = 2004$ } you hear beats with beat freq $|f_1 - f_2| = 4$ Hz.

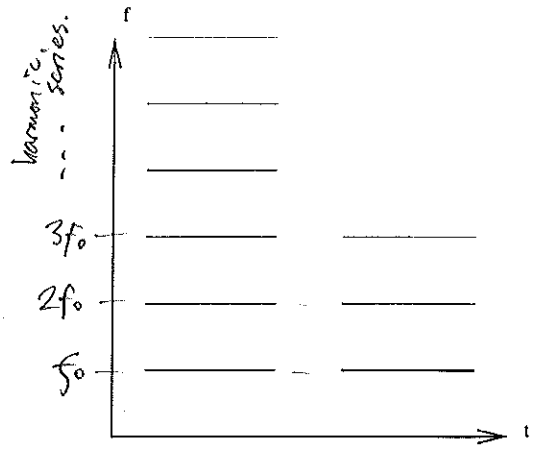
The tone which appears to be beating (varying amplitude) is at the average freq $\dfrac{f_1+f_2}{2} = 2002$ Hz.

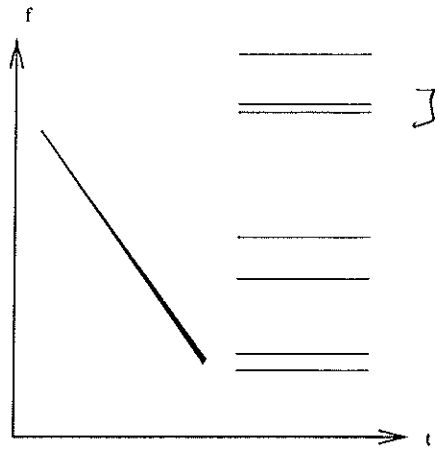
Sketch a graph of the combined signal:



See lecture notes for derivation.

4. Describe in as much detail as you can what sounds these two spectrograms correspond to (discuss periodicity, pitch, timbre, etc)





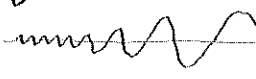
→ probably dissonant.

2 notes are sounding, with the same pitch but different timbre (harmonic content).

The pitch is clearly the same because the partials form a harmonic series in both cases, with the fundamental f_0 being the same.

Both are periodic signals since such signals have partials in a harmonic series. But their c_1, c_2, c_3 etc. coefficients are different.

The first will be harsh, the second more mellow.

A pure tone with frequency decreasing but amplitude increasing, for instance the graph could be  .

This is followed by a bell-like sound (or chord), which is not a periodic signal (since the partials are not in a harmonic series). It may not have a well-defined musical pitch (there's no common divisor). It may be dissonant since there are partials near each other (within 10% in frequency).