Barnett.

~ SOLUTIONS ~

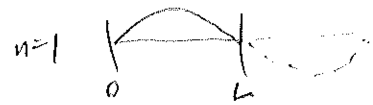
Math 5: Music and Sound. Fall 2010. Quiz 2.

35 mins (3 questions worth about equal points)

Please write on this paper, show your working. The last page has useful information.

1. A violin string is 0.3 m long.

2. (a) If waves travel at speed 240 m/s on the string, what frequency is the fundamental string vibration mode?



$\lambda = 2L$ wavelength on string.

so $f_1 = \frac{c_{string}}{\lambda} = \frac{c_{string}}{2L} = \frac{240}{2(0.3)} = 400\text{Hz}$

2. (b) Explain how the tension would need to be changed by the player to *lower* this fundamental pitch by an *octave*.

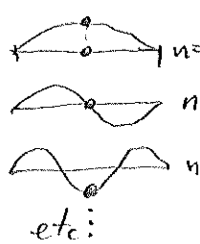
want $f \to \frac{1}{2}f$ ie $c_{string} \to \frac{1}{2}c_{string}$.

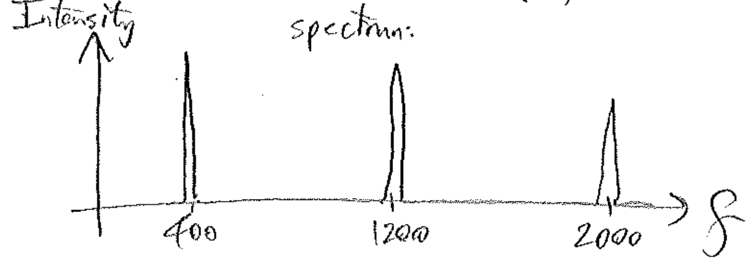
But $c_{string} = \sqrt{\frac{T}{\rho}}$ so halve c_{string} need $T \to \frac{1}{4}T$, ie 4 times less tension.

3. (c) The string is plucked exactly half way along. Draw a spectrum that could be produced, labeling frequencies of the lowest three partials heard. ← $x_p = L/2$.

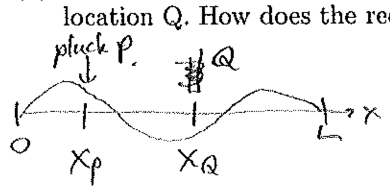
Excitations $\alpha_n = u_n(x_p) = \sin\frac{n\pi x_p}{L} = \sin\frac{n\pi}{2} = 1, 0, -1, 0, \dots$ (n=1) etc.

Or.





(+1) (d) BONUS: The string is instead plucked at general location P and an electric pickup is used at location Q. How does the recorded timbre change if Q and P are swapped? Why?



$\alpha_n = u_n(x_p)$ for each mode number n.

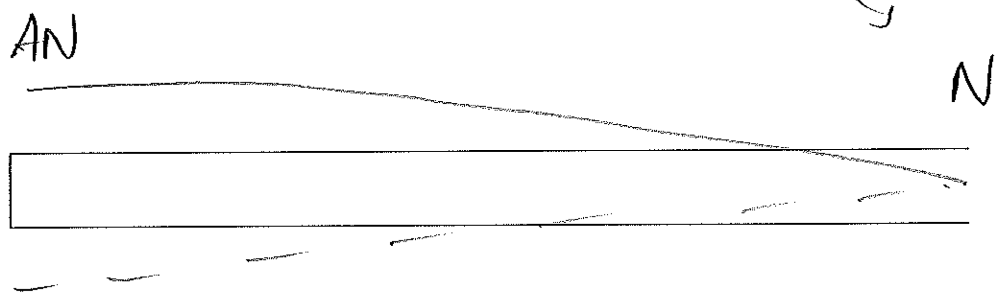
but pickup gets amplitude $c_n = \alpha_n u_n(x_Q)$
$= u_n(x_p)\, u_n(x_Q)$

This c_n unchanged if P↔Q swapped. ⇒ no change.

2. (a) Sketch graphs of pressure amplitude vs position along the closed-open pipe shown, and label nodes and antinodes:

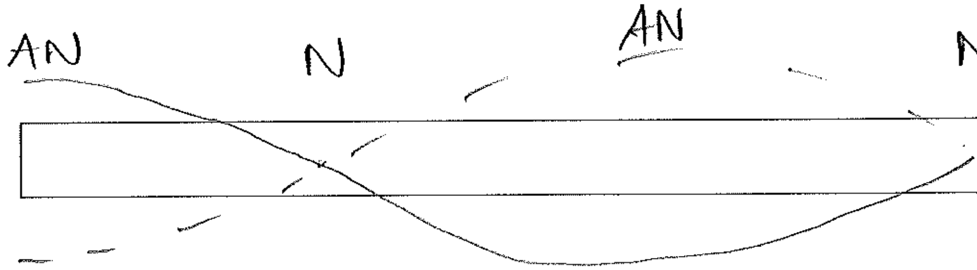
remember pressure ∝ 0 where pipe opens to air.

mode 1



2

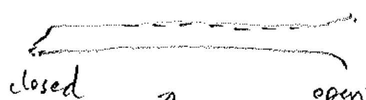
mode 2



(↑ widen here in part (c))

(b) A contrabass clarinet (closed-open) is 3.0 m long (it's folded! Assume this is the effective length). Find the frequencies of the first two partials it produces.

3



(assume all fingerholes are closed !)

$n=1$ wavelength $\lambda = 4L$.

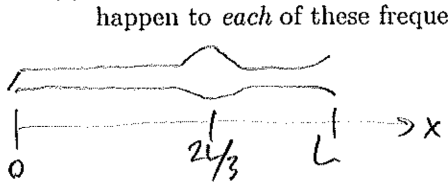
so $f_1 = \frac{c}{\lambda} = \frac{c}{4L} = \frac{340}{4(3)} \approx 28.3\,Hz$ (low!)

$n=2$ $\lambda = \frac{4}{3}L$ since there's 3/4 a full wave in pipe.

$\Rightarrow f_2 = \frac{3c}{4L} = 3f_1 \approx 85\,Hz$

(c) If the clarinet tube were widened 2/3 of the way along the pipe from the mouthpiece, what would happen to *each* of these frequencies?

2



rule: widen at {node / AN} → {higher / lower} mode freq.

$n=2$ mode has AN at $2L/3$ ⇒ f_2 will drop.

$n=1$, $2L/3$ a bit closer to node than AN ⇒ f_1 rises (only a little)

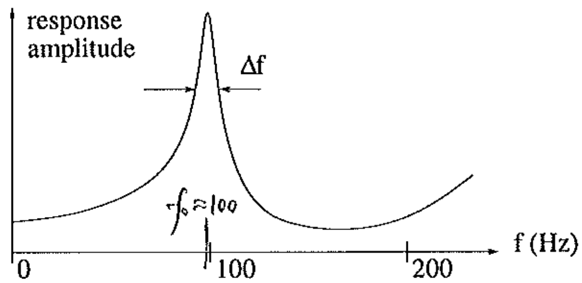
(d) BONUS: Where would you ideally place the register key on a clarinet and why?

2008 only.

Register key causes $n=1$ mode to be damped but not $n=2$, so reed excites $n=2$ instead of the usual $n=1$.

$x = L/3$ is where $n=2$ has N so will not be damped; this is a good location.

3. A microphone is placed inside the body of a guitar of volume 6 liters (0.006 m^3) while a pure tone is sounded outside (ignore the strings). The amplitude measured depends on the pure tone frequency as follows:



2 (a) If the frequency width Δf is 5 Hz (just under one semitone) compute the Q factor of the lowest resonance.

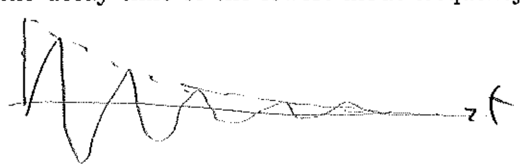
not needed.

$$f_0 = 100 \qquad \frac{\Delta f}{f_0} = \frac{1}{Q}$$

$$\text{so} \quad Q = \frac{f_0}{\Delta f} = \frac{100}{5} = 20$$

easy.

2 (b) Compute the decay time of the lowest mode frequency when the air hole in the guitar body is slapped.

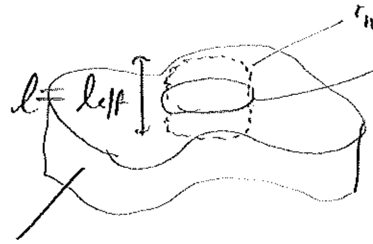


decay time τ

$$Q = \pi f_0 \tau \quad \text{so} \quad \tau = \frac{Q}{\pi f_0} = 0.0637\ s.$$

(short, only ~6 periods).

2 (c) If the hole in the guitar body has area 20 cm^2 (careful, not m^2!) what is the (effective) length of the 'neck' or plug of air which oscillates in and out of the hole?



$V = 0.006\ m^3$

$a = 20 cm^2 = 0.002\ m^2$. since $1\ cm^2 = (0.01 m)^2 = 10^{-4} m^2$.

$$100 = f_{Helm} = \frac{c}{2\pi}\sqrt{\frac{a}{Vl}} \qquad \text{solve for } l.$$

$$\text{so} \quad l = \left(\frac{c}{2\pi f_{Helm}}\right)^2 \frac{a}{V}$$

$$= \left(\frac{340}{2\pi \cdot 100}\right)^2 \frac{0.002}{0.006} = 0.098\,m$$

$\approx 9.8 cm$.

About right since expect roughly spherical 'plug' of air if guitar body very thin.