Paper One: The Mathematics of the Pythagoreans

This paper is well composed in a comprehensive narrative form, and it is a good model for effective thesis development. The writer has revealed thesis elements clearly and early in the paper, has developed and extended these throughout the opening paragraph, and has concluded the first paragraph with a thesis summary. Examples of thesis development appear in blue below. The summary thesis statement, which appears in blue type and is underlined, reflects what has been said so far and predicts what is to come.

Pythagoras, the Greek philosopher and mathematician best known for the famous Pythagorean Theorem, was interested in much more than just the proof of $a^2+b^2 = c^2$. Rather, he and his followers--known as the Pythagoreans--also explored the *principles of* mathematics, the *concept of* mathematical figures, and the *idea of* a proof ("Pythagoras of Samos" 3). Their method of exploration and the motivation behind it was to understand and come in "contact with a higher, a divine reality" (DeVogel 196). As a result, there was a mixture of mathematics and mysticism in the Pythagoreans' geometry, arithmetic, astronomy, music, and theory of numbers (Hersh 92). Although the Pythagoreans were criticized by many who found fault with combining a higher truth with mathematics (Aristotle, for example, was critical of their confusion of the physical and metaphysical), it cannot be denied that they were the first thinkers in the Western world to mathematicize the universe. They interpreted the world "in terms of order and symmetry, based on fixed mathematical ratios" and "maintained that the cosmos was expressible in terms of number" (Hersh 237; Philip 79). Due to this belief, and

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[1] Quote comes from: Hersh, Reuben. What is Mathematics, Really? New York: Oxford University Press, 1997

[2] Philip, J.A. Pythagoras and Early Pythagoreanism. Canada: University of Toronto Press, 1966. page 60.

[3] Weisstein, Erik W. *Pythagoras of Samos*. 1996-1999. http://www.astro.virginia.edu/~eww6n/bios/Pythagoras.html.

motivated by their mysticism, the Pythagoreans uncovered such ideas as the five regular solids, the Pythagorean Theorem, the existence of irrational numbers, and many other key ideas (Guthrie 329-330). Thus, by searching for a divine truth and universal harmony through mathematics, the Pythagoreans made many important discoveries in different areas of math and science that are both mathematically sound and continue to be applied today.

The Pythagoreans made numerous findings in the field of astronomy that are surprisingly accurate, considering the lack of observational instruments in their day. For example, they hypothesized that the earth is round by observing the shadow of the earth on the moon during a lunar eclipse. Also, they believed that the sun and other planets are spheres as well, and that they orbited each other and a "central fire" in a circular motion. (Weisstein 1). Furthermore, the Pythagoreans recognized that the "orbit of the moon was inclined to the equator of the Earth and that both the morning and evening star were the planet Venus ("Pythaogras of Samos" 5; Weisstein 1). The metaphysical part of their astronomy relates to their belief in harmony. DeVogel writes: "[the] basic principle of a mathematically ordered universe [is that] all order is based on numerical proportion; that is why harmony is the universal, cosmic law" (163). An example of this belief is the Pythagoreans' conjecture of the existence of ten planets. They could only observe nine, but hypothesized the existence of a tenth planet, a "counter-earth"--to them the number ten symbolized perfection and unity; due to their belief that the universe was harmonical, there had to be ten planets in order to fit their perception of the universe (DeVogel163; Philip 78). Also, the Pythagoreans believed that the planets generated sounds while orbiting the universe, producing a "harmony of the spheres" (Weisstein 1). This leads into Pythagoras' work on music and music theory.

Pythagoras of Samos. http://www.intelligentchild.com/astronomy/pythaogras.html.

Another great discovery attributed to Pythagoras is that "musical intervals may be

reduced to numerical ratios" (DeVogel 162). He discovered three musical intervals, the fourth, the fifth, and the octave by experimenting with the monochord, a one-stringed instrument with a moveable bridge (Philip 126, Guthrie 24). Pythagoras observed that vibrating strings produce harmonious sounds when the "ratios of the lengths of the strings are whole numbers" ("Pythaogras of Samos" 4). Specifically, the ratio of the octave is 1:2; the perfect fifth, 2:3; the perfect fourth, 3:4. (Guthrie 29). This mathematical and musical discovery is linked by the Pythagoreans to the universe itself; they considered the universe to be composed of numbers and thus concluded that the ratios that Pythagoras discovered in music were also present in the relations of celestial bodies to one another. As mentioned before, the Pythagoreans believed that the planets produced sounds while orbiting; furthermore, they applied the newfound ratios to the harmonious sounds that the planets supposedly produce (Philip 123). Again, this is another instance of applying mathematics to the universe, albeit metaphysically. More importantly, however, is that the discovery of musical intervals was the foundation of music theory and continues to be relevant.

The mathematical aspect of Pythagoreanism that is most metaphysical and subject to criticism is number theory—the belief that there is a deeper, divine significance to numbers. According to the Pythagoreans:

> …number was the principle of a divine order in the Universe. The study of number and its laws therefore was the immediate contemplation of the divine Law by which everything is held together. (DeVogel 196)

The Pythagoreans attributed varying qualities to different numbers, each one being a "self-subsistent entity having a character and property of its own" (79). Also, the way in which numbers were combined was significant; for example, justice was considered to be a reciprocal relation between crime and punishment as well as a number squared (Hersh 97).

A table of the integers from one to six and their meaning is seen below:

Integer	Meaning[5]
1	generator of numbers and number of reason
2	first even or female number
3	first true male number
4	justice or retribution
5	number of marriage (union of male and female)
6	number of creation

Most important is their belief that numbers were physical, existing entities that were the physical components of everything in existence. As Philip writes: "Everything is number" (61). They did not believe or reason that a number is just a number, for purposes of quantification. Nor did the Pythagoreans distinguish between the ideal and mathematical number (DeVogel 197; 204). Rather, their use of numbers was a search for truth and patterns in the universe; this search led them into true discoveries in mathematics and geometry.

"The Pythagoreans…were the first to use numerical and geometrical diagrams as models of cosmic wholeness and celestial order" (30). For example, they represented the number ten in the geometric form of the tetraktys (see figure below).

The Pythagoreans' method of arranging numbers geometrically may actually have been the way by which they discovered geometrical theorems (29). Another instance of arranging mathematical numbers into geometrical forms can be seen below:

Illustration here

The Pythaogoreans are also credited with the construction of the five regular solids. Pythagoras probably constructed them out of right triangles (see below). Predictably, each solid had a symbolic meaning in relation to the Universe (314).

Solid	# of Sides	Symbolic Meaning
Cube	6	Earth
Tetrahedron	4	Fire
Octahedron	8	Air
Dodecahedron	12	Aither
Icosahedron	20	Water

From Hersh page 93.

The Pythagoreans also tackled the idea of dimensions in terms of geometry. One represented the point . Two represented the line . Three represented the surface, and four was the tetrahedron , the first three-dimensional form (Guthrie 29). The single point was the generator of dimensions, the two points were a line of the first dimension, the three points not on a line were a triangle of the second dimension, and the four points not on a plane, the tetrahedron, had a volume of dimension three (Hersh 93). The Pythagoreans experimentation and formation of geometrical forms led them into significant mathematical discoveries because they recognized that numbers and geometry were linked.

As mentioned before, because the Pythagoreans tended to arrange numbers into geometrical shapes, they were inclined to make discoveries in the mathematical field of geometry. For example, they proved that the sum of the interior angles of a triangle is equal to 180 degrees by the following formula:

A polygon with n sides has sum of interior angles 2n-4 right angles.

Ex: a triangle has 3 sides

2(3)-4 = 2

2(90°) = 180°

This generalization is applicable to all polygons, in fact. It remains relevant, in our day, for purposes of construction, engineering, and applications of geometry. Also, they were aware of the properties of similar figures, and, as mentioned above, the five regular solids (Guthrie 329-330).

The Pythagoreans uncovered the existence of irrational numbers, as well. The concept of irrationality was anathema to a group of people who believed that numbers were commonsensical and upheld a harmonious universe, and yet they acknowledged and proved the irrationality of the √2. Again, they uncovered the irrationality through geometric means; they discovered that the diagonal of a square was incommensurable with its side[6]. Mathematically, this conjecture can be demonstrated by showing that there

is "no pair of integers x and y, such that $(x/y)^2 = 2$ (81) The Pythagoreans' most famous discovery, however, was the Pythagorean Theorem. According to Euclid, the theorem states: "In any right triangle, the sum of the squares of the lengths of the two shorter sides equals the square of the length of the long side" (Hersh 272). Again, the Pythagoreans did not think of this in purely abstract means; they actually constructed squares on the sides of the triangles and calculated the areas of the squares, finding that the sum of the area of the two smaller squares was equal to the area of the larger square. This is in keeping with their geometrical way of viewing numbers and mathematics (apart from viewing the world metaphysically). See below for proof.

proof here

[6] http://www.scsd.k12.ny.us/levy/math/Pythagoras2.html

Although the Pythagorean theorem was known to the Babylonians 1000 years earlier, Pythagoras was probably the first to prove it ("Pythagoras of Samos" 4). Due to this monumental proof, the idea can be applied to, among other things, construction (the pitch of a roof, the determination of whether or not a wall was perpendicular to the floor, et cetera), mathematics (such as the distance between two points on a graph), astronomy (measuring distances to and between stars), archaeology (measuring sites) and so on. The Pythagorean Theorem and other discoveries of the Pythagorean school are applied and relevant from the time the discoveries were made to the present day.

Thus, we see that, through Pythagoras' and his followers' search for truth in numbers and their formation of numbers into geometrical forms, many key mathematical ideas were uncovered that serve humans' purposes to this day. Although the Pythagoreans were motivated less by a purely abstract quest for knowledge than motivated by a metaphysical desire to explain the universe, they nevertheless did uncover certain mathematical realities. As Aristotle wrote: "…the thinkers known as the Pythagoreans were the first to pursue mathematical studies and advance them" (Philip 78). As mathematical metaphysicists, they contributed many important ideas to mathematics. Most of this concluding paragraph, and the one before it, is vague and, at times, awkward in construction. I think the writer might have become tired by the time s/he arrived at this point in the paper and fell back on awkward, repetitious word usage and sentence structure. This is disappointing, because throughout most of the paper the writing is smooth and pleasant to read. I have noted numerous errors, but most of these are minor and could have been revised easily. Falling away at the end of the paper is common among student writers. Weak endings diminish the quality of papers; the last writing needs to be as strong as the first. If there is nothing more to say, don't say it. Simply write a succinct conclusion and stop. Old soldiers may never die, may just fade gloriously away; papers that fade away just die.

Additional comments follow works cited page.

Works Cited

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I have criticized this paper relentlessly. I have plumbed it for problems that you might not repeat in your own writing. What follows is a summary of the good, the bad, and the ugly.

The Good:

~ The thesis is stated early in the paper.

~ The thesis is not ambiguous.

~ The writer carries the thesis throughout the paper.

~ The writer has provided topic sentences at the start of paragraphs in which s/he presents new ideas, facts, and/or examples.

~ Sentence structure is often commendable.

~ Paragraphs usually build on information previously provided and support their topics, as well as the overall thesis.

~ The writer has used direct quotes judiciously and has, for the most part, integrated quotes effectively.

~ The paper is well cited, and citations are properly structured and punctuated—most of the time. (I have a brief PowerPoint presentation on integrating quotations. I will email it to students upon request.)

~ The writer includes examples that support statements and conclusions. S/he integrates these into the text effectively.

The Bad:

~ Some sentences are not parallel in structure.

~ The writer has used a style of citation that is inconsistent in form; it seems to be a mix of more than one style. Choose a style and stay with it.

~ The writer suffers from vague pronoun reference syndrome. I can provide help for others who are similarly afflicted. Readers, professors in particular, become cranky when they are forced to search for antecedents. I don't think we want the person who grades the papers to do so in a cranky state of mind.

~ The writing is wordy at times. We certainly and most definitely do not want to hand in and submit to our professor a paper that does not meet the requirements and instructions (such as length, topic, and deadline criteria) for a certain individual, and appropriate long-term writing assignment that has been described, discussed, and outlined on more than one occasion during several of our hour-long thrice weekly class meetings, as well as on the course website for this specific term of enrollment, by the above-mentioned professor, who teaches, and apparently wrote the book for, this particular course we are taking in the discipline of mathematics, which might or might not be our chosen and selected major, and who probably, in addition to our course, teaches many other courses at the college we are currently attending as students, and who perhaps also teaches, or has previously taught, at other colleges we will never attend (unless we do poorly at this one), for grading and assessment, most importantly since the score we earn and are given, based on the quality of our writing and wordsmithing, for the paper to which we have previously referred, is worth and will count for a significant and actually quite large, portion and percentage of the final and most definitive grade we receive when the winter term now in session comes to a close and finally ends, and this course simultaneously at that same time also comes to a close.

OR

We want to earn A’s on our Math 5 midterm papers.

The Ugly I looked for something ugly in this paper and found nothing.