Homework 2: Due Wednesday, April 16

Section 3.2: #6, 10, 13, 18, 34a

Section 2.2: #12, 14

Problem 1: An *anagram* of a word is a rearrangement of the letters. For example, $PLPEA$ is an anagram of $APPLE$. How many anagrams are there of MISSISSIPPI?

Problem 2: In poker, a *straight* is 5 cards which can be placed in ascending order in which no ranks are skipped. For example, the hand consisting of

- 8 of hearts
- 9 of clubs
- 10 of clubs
- Jack of spades
- Queen of hearts

is a straight. Suppose that an Ace can be considered as either having rank 1, or as having rank just above a king (but it is not allowed to "wrap around" from King, Ace, Two). What is the probability of getting dealt a straight?

Problem 3: Recall that a *flush* in poker is a hand in which all five of your cards have the same suit. Suppose that you are playing a game of poker in which each 2 is a "wild card". That is, you can take each 2 to represent any other card. For example, if you have three hearts, the 2 of spades, and the 2 of diamonds, then this would be considered a flush because we can pretend that the two 2's are other hearts. What is the probability of getting dealt a flush? How much more likely is this than the probability of getting dealt a flush when there are no wild cards?

Problem 4: By the Binomial Theorem, we know that for every real number x and every natural number n we have

$$(x+1)^n = \sum_{k=0}^{n} \binom{n}{k} x^k$$

Use this to show that

$$\binom{n}{1} + 2\binom{n}{2} + \cdots + n\binom{n}{n} = \sum_{k=1}^{n} k\binom{n}{k} = n2^{n-1}$$

for every n.

Problem 5: Use Stirling's Approximation to $n!$ to show that

$$\lim_{n\to\infty} \frac{b(2n, \frac{1}{2}, n)}{\frac{1}{\sqrt{\pi n}}} = 1$$

In other words, if $f(n)$ is the probability of getting exactly n heads when we flip a fair coin $2n$ times, then the function f behaves very much like the function $g(n) = \frac{1}{\sqrt{\pi n}}$ for large values of n.