

HOMEWORK VII

ALGEBRAIC COMBINATORICS (MATH 68)

Due October 30, 2019, at the **beginning of the class**

Collaboration among students to find key to the solution is encouraged, but each person must write the homework in his/her own words. You must write the name of the students with whom you work for each problem, as well as any written resource (web, book, etc.) that has been extensively used.

You must write the appropriate justification as part of the solutions.

- (1) Compute the product of $t = \begin{array}{|c|c|c|c|} \hline 1 & 2 & 2 & 3 \\ \hline 2 & 3 & 5 & 5 \\ \hline 4 & 4 & 6 & \\ \hline 5 & 6 & & \\ \hline \end{array}$ and $u = \begin{array}{|c|c|} \hline 1 & 3 \\ \hline 2 & \\ \hline \end{array}$ by rectifying $\begin{array}{|c|c|c|c|c|c|} \hline & & & & & 1 & 3 \\ \hline & & & & & 2 & \\ \hline 1 & 2 & 2 & 3 & & & \\ \hline 2 & 3 & 5 & 5 & & & \\ \hline 4 & 4 & 6 & & & & \\ \hline 5 & 6 & & & & & \\ \hline \end{array}$.

- (2) Use the claims in the handouts from Monday to prove the associativity of the product of tableaux.
- (3) A permutation is said to avoid a *pattern* $p_1p_2 \dots p_n$ if it does not contain a subpermutation whose elements are placed in the same order as p_1, p_2, \dots, p_n . For example, 3412 avoids 213.
- (a) Describe the set of shapes for insertion tableaux for 321-avoiding permutations.
- (b) Describe the set of shapes for recording tableaux for 12-avoiding permutations.
- (4) What are the P - and Q -tableaux associated with permutation 328951647.

- (5) What permutation do you get from the P -tableau $\begin{array}{|c|c|c|} \hline 1 & 2 & 6 \\ \hline 3 & 5 & 7 \\ \hline 4 & 8 & \\ \hline 9 & & \\ \hline \end{array}$ and Q -tableau $\begin{array}{|c|c|c|} \hline 1 & 4 & 5 \\ \hline 2 & 6 & 7 \\ \hline 3 & 8 & \\ \hline 9 & & \\ \hline \end{array}$?

- (6) Verify that the hook formula computes the right number of standard Young tableaux of shape $(l, 1, \dots, 1)$ (there are m rows). What is that number? (Here you should compute it in two different ways, and one of them must be with the hook formula.)
- (7) If σ has $n - 2$ fixed points, what can be said about P and Q ? I'm expecting two things.

(8) If $Q =$

1
2
3
4
5
6
7

, what is P ? What is σ ? How do you know?

Good luck!