

Mathematics 69  
Winter 2023  
Overview

Assignments and other course information can be found on the Canvas page.

Mathematical logic, in the sense of this course, applies mathematical tools to model and study the activity of working mathematicians.

Working mathematicians prove theorems (among other things). We will study formal deductions in a formal language, which are proofs formulated in a precise framework, so precise that they can be studied mathematically: We can prove things about them. Our results about formal deductions can illuminate the proofs found in mathematicians' research papers and in mathematics students' homework papers.

We will begin the course with the study of the language of sentential logic, a formal language that is not powerful enough to model the language used by working mathematicians. Building on this, we will study the more powerful language of first order logic, and the notion of formal deduction in first order logic. We will prove Gödel's Completeness Theorem, a major result, which states that this notion of formal deduction completely captures the power of mathematical proof in the following sense: Given any axiom system (for example, the axioms for a real vector space) and any statement in our language, either that statement can be proved from the axioms by means of a formal deduction, or else we cannot hope to prove the statement because there is a context (in our example, a real vector space) in which the axioms are true but the statement is false.

We will also see the proof of Gödel's Incompleteness Theorem. The Incompleteness Theorem is one of the most important mathematical results of the twentieth century, with important practical and philosophical consequences for mathematics.

An important philosophical consideration through our study will be the notion of "effective" or "algorithmic" process. For example, Gödel's Incompleteness Theorem will tell us that there is no algorithm to list all the true statements about the natural numbers. In other words, it is impossible to program a computer to answer all possible questions (even in our restricted formal language) about the natural numbers.

Prerequisite for this course: Experience with mathematical structures and proofs, as offered by such courses as Mathematics 71, 54, or 24. If you are unsure about your preparation, please talk to Professor Groszek.

Math 69 satisfies the culminating experience requirement for mathematics majors.

Math 69 is appropriate for any graduate student who wants to take a course in logic.

### Course Structure

Instructor: Marcia Groszek <marcia.groszek@dartmouth.edu>.

Textbook: A Mathematical Introduction to Logic (second edition) (ISBN: 978-0122384523) by Herbert Enderton.

Scheduled Classes: 2 period (MWF 2:10-3:15 EST, x-hour Th 1:20-2:10 EST), 28 Halde-  
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We will occasionally, but not regularly, use the x-hour. Because of Martin Luther King Day, instead of meeting on Monday 1/16, we will meet during the x-hour on Thursday 1/19.

I request that everyone wear masks in the classroom.

Office Hours: Tuesday 1:30-3:00 and Friday 12:00-1:30, and Monday evening on Zoom. You can also email me for an appointment at another time.

Scheduled office hours are always drop in; you need not make an appointment. If you have a conflict with regularly scheduled office hours, or prefer a one-on-one meeting, you can always make an appointment for another time. Please come to office hours whenever you are inspired to, and whenever you suspect you ought to. Bring questions about the class, about the homework, about mathematical logic, about studying mathematics, about graduate school... You are always welcome.

Grades: The course grade will be based upon the midterm exams, the homework, and the final paper, each counting for about one third of your grade. In borderline cases, factors such as class participation, demonstration of the ability to work independently and collaboratively, and a steady record of improvement will be considered.

Exams: There will be two midterm exams. Both will be open-book and untimed. Tentative due dates are Monday January 30 and Monday February 20. See the Policies section for how the honor principle applies to exams.

Homework: There will be two kinds of homework, reading assignments, which include a written component, and graded written homework. See the Policies section for how the honor principle applies to written homework.

Reading assignments are due at noon on the class day following the day they are assigned; for example, reading assigned on Friday is due on Monday, and reading assigned on Monday is due on Wednesday. Reading assignments are 25% of the homework grade. You get full credit for turning in the assignment on time. There is no credit for late reading assignments, but I will automatically excuse up to 5 late or missing reading assignments.

Graded written homework is due at noon on Wednesday of the week following the day it is assigned. Homework late by at most one week will count for partial credit: 90 percent if turned in by 5:00 on the day it is due, 80 percent if turned in by noon on the day of the next class, 50 percent if turned in within one week of the due date. Remember that your homework is being received by a computer; a few seconds late is still late.

Final Paper: For your final paper, you will choose one of several topics that connect mathematical logic with other areas of mathematics. The final paper assignment will describe these topics, including, for each topic, a list of questions your paper should address. The assignment will also describe how the honor code applies to the final paper.

Your final paper must be typed (or word-processed). The best way to do this is to use LaTeX. We will use an x-hour or two to introduce LaTeX.

Classes: There will be a reading assignment due each class, which I will assume you have read. Come prepared to ask questions; I will come prepared to answer them. If nobody asks questions, I will assume everyone understood the reading.

In addition to answering questions, I may give a brief lecture supplementing or expanding on the reading.

Much of our class time will be spent working in small groups (in breakout sessions), and then coming together as a class to review our work. The questions you work on in these sessions are not to be turned in as homework (unless they also appear in a homework assignment). However, writing them up for your own notes is a good idea.

## Policies

The following are mostly College policies. Please talk to me if you have any questions or difficulties about these or any other issues.

### Academic Integrity:

Academic integrity is at the core of our mission as mathematicians and educators, and I take it very seriously. I also believe in working and learning together.

Here is a statement of the Academic Honor Principle, and here are some additional resources about standards of conduct. The booklet Sources and Citations at Dartmouth can be a useful guide.

In this class, you are encouraged to work together on homework, and you may consult outside sources. but you must write up your solutions yourself. Even if a group of students works together to produce a proof, each student must separately write up their own version of that proof. The solutions you submit must be written by you alone and represent your own understanding. Any copying (electronic or otherwise) of another person's solutions, in whole or in part, is a violation of the Academic Honor Code.

Moreover, if in working with someone they have provided you with an important idea or approach, they should be explicitly given credit in your writeup. Crediting another person will not lower your grade, which is based on your demonstrated understanding, not on how you gained that understanding. Hints given in office hours need not be cited.

On exams, you may not give or receive help from anyone. You should discuss the exams only with the instructor, for clarification of problems. You may use your textbook, your own notes and homework, and any materials distributed in class. You may not use outside sources, including but not limited to other textbooks and online sources. If you have any questions as to whether some action would be acceptable under the Academic Honor Code, please speak to me, and I will be glad to help clarify things. It is always easier to ask beforehand.

The final paper assignment will include information about how the honor principle applies to your final paper.

### Religious Observances:

Some students may wish to take part in religious observances that occur during this academic term. If you have a religious observance that conflicts with your participation in the course, please speak with me before the end of the second week of the term to discuss appropriate accommodations.

### Schedule Conflicts:

Some students may wish to take part in extracurricular activities or have other personal commitments that conflict with class time. If you have a conflict, please speak with me before the end of the second week of the term.

#### Student Accessibility and Accommodations:

Students requesting disability-related accommodations and services for this course are encouraged to schedule a meeting with me as early in the term as possible. This conversation will help to establish what supports are built into my course. In order for accommodations to be authorized, students are required to consult with Student Accessibility Services (SAS, 603-646-9900; see the Getting Started with SAS webpage) and to request an accommodation email be sent to me. We will then work together with SAS if accommodations need to be modified based on the learning environment. If students have questions about whether they are eligible for accommodations, they should contact the SAS office. All inquiries and discussions will remain confidential.

#### Diversity & Inclusion:

I strive for a learning environment that supports diversity of thought and experience and honors everyone's identity. Please tell me if anything makes you uncomfortable. As a participant, please respect your classmates and support our commitment to diversity.

#### Mental Health and Wellness:

The academic environment at Dartmouth is challenging, our terms are intensive, and classes are not the only demanding part of your life. There are a number of resources available to you on campus to support your wellness, including your undergraduate dean, Counseling and Human Development, and the Student Wellness Center. I encourage you to use these resources to take care of yourself throughout the term, and to come speak to me if you experience any difficulties.

#### Title IX:

The Sexual Respect Website at Dartmouth provides a wealth of information on your rights with regard to sexual respect and resources that are available to all in our community.

Please note that, as a faculty member, I am obligated to share disclosures regarding conduct under Title IX with Dartmouth's Title IX Coordinator. Confidential resources are also available, and include licensed medical or counseling professionals (e.g., a licensed psychologist), staff members of organizations recognized as rape crisis centers under state law (such as WISE), and ordained clergy.

Should you have any questions, please feel free to contact Dartmouth's Title IX Coordinator or the Deputy Title IX Coordinator for the Guarini School. Their contact information can be found on the sexual respect website.

#### Textbook Costs and Financial Difficulty

If you encounter financial challenges related to this class, please let me know. You will need to use the textbook for this course.

#### Learning Resources:

Academic Skills Center (ASC): Open to the entire Dartmouth community, the ASC assists students in achieving their academic goals through tutoring and learning skills trainings.

Research Center for Writing, and Information Technology (RWIT): RWIT is a free service dedicated to helping members of the Dartmouth community develop more effective strategies for generating and organizing their ideas, finding and evaluating research sources, and presenting and revising compositions in a variety of media.

Dartmouth College Library: The Dartmouth College Library provides support through subject area specialization, course materials and reserves, reservable learning spaces, workshops & classes for students, research, scholarly publication, copyright, media, book arts, and more. The Library's Research Guides by subject area and your discipline's subject librarian are sources of specialized help for your course and students.

Note

In the Course Summary on Canvas, items are listed on the date they are due. Times are in Eastern Standard Time, and 12 pm means noon, not midnight. The item title generally says what date it is assigned. I encourage you to do graded homework when it is assigned, rather than wait until just before it is due.

Online classes, privacy, and intellectual property:

Math 69 will be held in person. Some of these policies will be relevant only if we need to move to online meeting for some reason.

By enrolling in this course, you accept and agree to the College's Consent to Record Statement that follows this short discussion.

Only the instructor may record any Math 69 meetings, including class sessions, breakout rooms, and office hours. I do not plan to record any Math 69 classes or office hours, and I will inform you if this changes for some reason.

Consent to Record Statement

1. Consent to recording of course and group office hours

a) I affirm my understanding that this course and any associated group meetings involving students and the instructor, including but not limited to scheduled and ad hoc office hours and other consultations, may be recorded within any digital platform used to offer remote instruction for this course;

b) I further affirm that the instructor owns the copyright to their instructional materials, of which these recordings constitute a part, and distribution of any of these recordings in whole or in part without prior written consent of the instructor may be subject to discipline by Dartmouth up to and including expulsion;

c) I authorize Dartmouth and anyone acting on behalf of Dartmouth to record my participation and appearance in any medium, and to use my name, likeness, and voice in connection with such recording; and

d) I authorize Dartmouth and anyone acting on behalf of Dartmouth to use, reproduce, or distribute such recording without restrictions or limitation for any educational purpose deemed appropriate by Dartmouth and anyone acting on behalf of Dartmouth.

2. Requirement of consent to one-on-one recordings

By enrolling in this course, I hereby affirm that I will not under any circumstance make a recording in any medium of any one-on-one meeting with the instructor without obtaining the prior written consent of all those participating, and I understand that if I violate this prohibition, I will be subject to discipline by Dartmouth up to and including expulsion, as well as any other civil or criminal penalties under applicable law.

Final Paper Resources:

Here you can find some guidelines for citing and acknowledging sources. The Institute

for Writing and Rhetoric in Baker Library provides a number of resources, including peer tutors.

Here is a guide to writing in mathematics courses; the intended audience is first term calculus students who are writing rather short papers describing the solutions to calculus problems.

Here is a paper about writing mathematics; the intended audience is undergraduate mathematics majors writing serious mathematics papers.

You can find the final paper assignment, and an example of an expository mathematics paper, in the course files.