

Math 6
Worksheet 3
Due Monday April 19

Please discuss the following questions with your assigned groups. You may take notes on all items you discuss with your classmates, however you are to write up your solutions independently of one another and without assistance. Your solutions should be written up carefully and neatly on a separate sheet of paper. You should write in **complete sentences** and explain all steps taken and tools used (such as theorems or results from class) in reaching your final answers. Please also include at the top of your write-up a list of people with whom you discussed these problems.

- 1) A box contains 3 black and 2 white hats. Three people standing in a line are facing forward and each can see only the person or persons in front of her. Each of the three people had one of the hats placed on her head but does not see which color. Each person can, however, see any hats in front of her and each knows what hats were in the box to begin with. When the person in the rear is asked what color her hat is, she replies "I don't know." When the person in the middle is asked, she answers, "I don't know." When the person in front is asked, she responds "I know what color my hat is." What color is it and how does she know? Please explain your reasoning.

(Next week, after we've discussed valid arguments, you'll be asked to turn this reasoning into a valid argument!)

In class this week, we talked about what it means for two statements to be logically equivalent. Logical argument is based not only on logical equivalences, but also on statements that logically imply others. What does it mean for one statement to logically imply another? It is defined as follows:

Definition: Given compound statements P and Q , we say that P *logically implies* Q (and we write $P \Rightarrow Q$) whenever $P \rightarrow Q$ is a tautology.

- 2) Use truth tables to show that

$$[(p \rightarrow q) \wedge (q \rightarrow r)] \Rightarrow (p \rightarrow r)$$

is a logical implication. This implication is called *hypothetical syllogism*.

A *syllogism*ⁱ is a series of conditional statements (i.e., “if... then”s) which we’ll call *premises*. By looking at pairs of premises and intermediate conclusions, we can repeatedly apply the logical implication of hypothetical syllogism (defined in question 2) to come up with one statement that is the conclusion of the syllogism. For example, let’s consider the following syllogism by Lewis Carroll:

1. Babies are illogical;
2. Nobody is despised who can manage a crocodile;
3. Illogical persons are despised;

It may be helpful to write out the premises in the form of “if... then” (*though you do not need to include this step in your write-up*):

1. If he is a baby, then he is illogical;
2. If he can manage a crocodile, then he is not despised;
3. If he is illogical, then he is despised;

(In using “he” here, assume we are referring to a single well-defined person.)

It may also be helpful to use symbols to represent the statements. Let a be “he is a baby”, b be “he is illogical”, c be “he can manage a crocodile”, and d be “he is despised”. We can now rewrite the syllogism as follows:

1. $a \rightarrow b$
2. $c \rightarrow \sim d$
3. $b \rightarrow d$

From premises 1 and 3, we can conclude that $a \rightarrow d$. We would next like to combine this with premise 3. First we may want to rewrite premise 3 as $d \rightarrow \sim c$ (the contrapositive of $c \rightarrow \sim d$). Now combining this with $a \rightarrow d$, we get $a \rightarrow \sim c$. So the conclusion of our syllogism is:

If he is a baby, then he cannot manage a crocodile.

Or, in more traditional English:

Babies cannot manage a crocodile.

3.) Find the conclusion for the following three syllogisms by Lewis Carroll:
(Be sure to use *all* of the premises!)

1. No ducks waltz;
2. No officers ever decline to waltz;
3. All my poultry are ducks;

1. There is no box of mine here that I dare open;
2. My writing desk is made of rosewood;
3. All my boxes are painted, except what are here;
4. There is no box of mine that I dare not open, unless it is full of live scorpions;
5. All my rosewood boxes are unpainted.

1. I despise anything that cannot be used as a bridge;
2. Everything, that is worth writing an ode to, would be a welcome gift to me;
3. A rainbow will not bear the weight of a wheelbarrow;
4. Whatever can be used as a bridge will bear the weight of a wheelbarrow;
5. I would not take, as a gift, things I despise.

ⁱ All syllogisms on this worksheet were taken from the website <http://home.earthlink.net/~lfdean/carroll/puzzles/index.html> and come indirectly from Lewis Carroll's text *Symbolic Logic*.