Math 6

Exam 1

Name: Key

Honor Principle: All work, up to and including final solutions, on this exam is to be your own. Calculators are permitted only for addition, subtraction, multiplication, division and squareroots.

Please show all your work.

Problem 0		5
Problem 1		20
Problem 2		15
Problem 3		20
Problem 4		15
Problem 5		15
Problem 6		10
Bonus		5
Total:		100

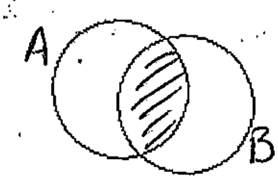
0. How did you study for this exam?

Any answer

1. For each of the following, label and shade the region of the Venn diagram representing the desired event, determine its size, and the probability of it occuring. (20 points)

(a)

$$\begin{aligned} n(A) &= 54 \\ n(B) &= 24 \\ n(A \cup B) &= 62 \\ n(U) &= 100 \end{aligned}$$

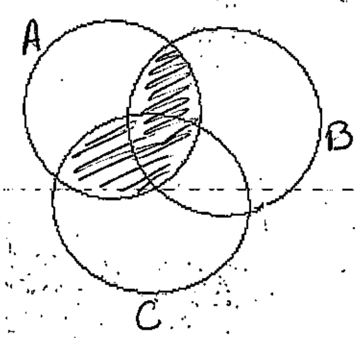


$n(A \cap B) = 16$
$P(A \cap B) = .16$

$A \cap B$?

(b)

$$\begin{aligned} n(A) &= 54 \\ n(B) &= 24 \\ n(C) &= 45 \\ n(A \cup B) &= 62 \\ n(A \cap C) &= 32 \\ n(B \cap C) &= 12 \\ n(A \cap B \cap C) &= 9 \\ n(U) &= 100 \end{aligned}$$

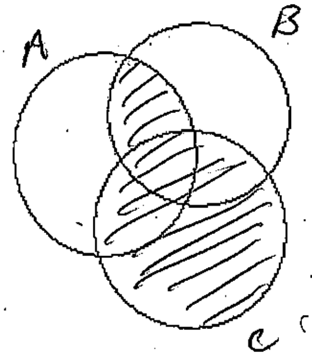


$n(A \cap (B \cup C)) = 39$
$P(A \cap (B \cup C)) = .39$

$A \cap (B \cup C)$?

(c)

$$\begin{aligned} n(A) &= 54 \\ n(B) &= 24 \\ n(C) &= 45 \\ n(A \cup B) &= 62 \\ n(A \cap C) &= 32 \\ n(B \cap C) &= 12 \\ n(A \cap B \cap C) &= 9 \\ n(U) &= 100 \end{aligned}$$

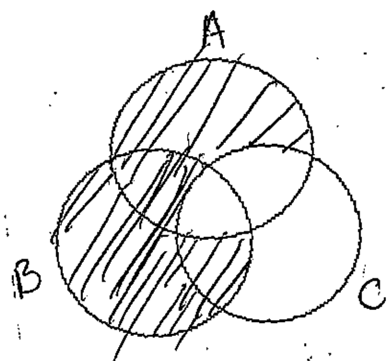


$n((A \cap B) \cup C) = 52$
$P((A \cap B) \cup C) = .52$

$(A \cap B) \cup C$?

(d)

$$\begin{aligned} n(A) &= 54 \\ n(B) &= 24 \\ n(C) &= 45 \\ n(A \cup B) &= 62 \\ n(A \cap C) &= 32 \\ n(B \cap C) &= 12 \\ n(A \cap B \cap C) &= 9 \\ n(U) &= 100 \end{aligned}$$



$(A \cap C') \cup (B \cap A')$

$n((A \cap C') \cup (B \cap A')) = 30$
$p((A \cap C') \cup (B \cap A')) = .30$

2. Find the probability of each of the following ice cream cones, given that there are 31 flavors of ice cream, 12 of which are fruit flavors and 10 of which are varieties of chocolate. (15 points)

(a) On a two scoop cone, both scoops are chocolate.

order matters, repitition allowed

$$\frac{10^2}{31^2}$$

(b) Three different flavors of ice cream in a three scoop cup.

order does not matter, repetition allowed

free problem: you do not have the tools necessary to solve

$$\binom{31}{3} \Big/ \binom{33}{2}$$

(c) One scoop of chocolate and one scoop of a fruit flavor on a two scoop cone.

order matters, no repetion numerator, repetition allowed denomenator.

$$\frac{2\binom{12}{1}\binom{10}{1}}{31^2}$$

2 is because chocolate can be top or bottom

(d) Two different flavors of chocolate and one scoop that is neither chocolate nor fruit in a three scoop cup.

order does not matter,
repetition allowed
free problem

$$\frac{\binom{10}{2}\binom{9}{1}}{\binom{33}{2}}$$

(e) A cone which has five different flavors, which, from top to bottom, are: chocolate, chocolate, fruit, vanilla, neither chocolate nor fruit. (There is only one vanilla.)

order matters

$$\frac{P(10,2)\cdot 12\cdot 1\cdot 8}{31^5}$$

$8 = 31 - \underset{\text{choc}}{10} - \underset{\text{fruit}}{12} - \underset{\text{vanilla}}{1}$

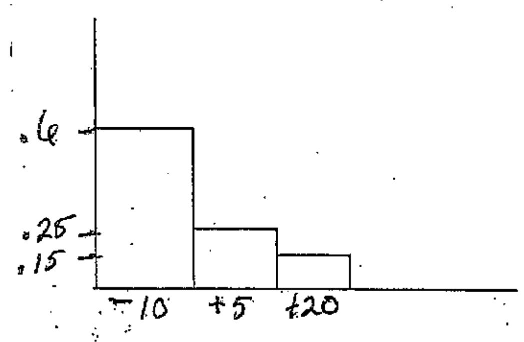
3. Find the expected value in the following situations. (20 points)

(a) The number of students in a sample of 400, when 10.2% of people are students.

$400(.102) = 40.8$

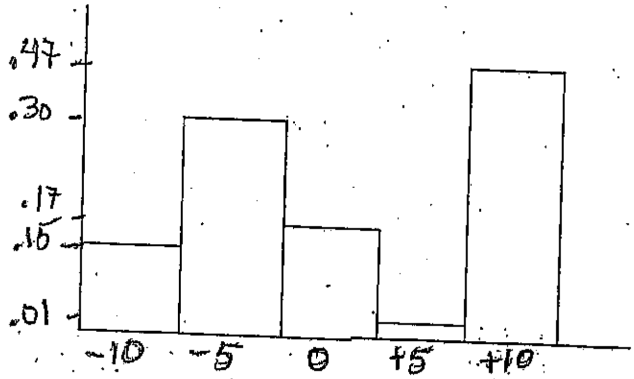
(b) The winnings in a game of chance with the following outcomes:

$(-10)(.6) + (5)(.25) + (20)(.15) = -1.75$



(c) The winnings in a game of chance with the following outcomes:

$$(-10)(.15) + (-5)(.30) + (0)(.17) + (5)(.01) + (10)(.47)$$
$$= 1.75$$



Note: probabilities add to 1.1 This is not allowed, in general, but here it is best to use the data given.

(d) The number of bad parts when any one of seven workers can make 50 parts (one worker makes all 50). The fail rates for each of the workers are:

Adam	2 of 50
Betty	5 of 50
Chris	7 of 50
Diane	1 of 50
Edmund	4 of 50
Francis	15 of 50
Gayle	9 of 50

Each worker has probability $\frac{1}{7}$ to make the parts.

$$\frac{1}{7}(2) + \frac{1}{7}(5) + \frac{1}{7}(7) + (\tfrac{1}{7})(1) + (\tfrac{1}{7})(4)$$
$$+ (\tfrac{1}{7})(15) + (\tfrac{1}{7})(9) = 6.14$$

4. Two ten sided dice are rolled sixteen times. The outcomes are 20, 13, 12, 14, 10, 12, 14, 2, 11, 8, 9, 15, 13, 10, 3, 13. Calculate five things which may be useful and say what they represent. (15 points)

① The mean is the outcome per trial if all trials had equal weight.

$\boxed{\bar{x} = 11.2}$

② The median is the outcome where there is an equal probability that the outcome would be higher or lower.

$\boxed{\text{median: } 12}$

③ The mode is the most frequent or most probable outcome.

$\boxed{\text{mode: } 13}$

④ The variance measures how different the numbers are.

$\boxed{\text{variance: } 18.93}$

⑤ The standard deviation measures how wide the spread of outcomes is.

$\boxed{s = 4.35}$

5. Suppose you have a weighted coin with probability $p = .56$ of landing heads. It is tossed 700 times. Find the approximate probabilities of the following events: (15 points)

(a) More than 400 heads.

For this problem, approximate with a normal distribution of mean $\mu = np = 392$ and standard deviation $\sigma = \sqrt{np(1-p)} = 13.1$

$p(x > 400) = \text{Pnorm}(z > 6.1) = 1 - .7291 = \boxed{.2709}$

(b) Between 300 and 350 heads.

$p(300 < x < 350) = \text{Pnorm}(z < -3.21) - \text{Pnorm}(z < -7.02)$

$= .0007 - 0 = \boxed{.0007}$

(c) More than 500 heads.

$p(x > 500) = \text{Pnorm}(z > 8.24) = 1 - 1 = \boxed{0}$

(d) Fewer than ~~315~~ 390 heads.

$p(x < 390) = \text{Pnorm}(z < -.15) = \boxed{.4404}$

(e) Between ~~340~~ 380 and ~~360~~ 400 heads.

$p(380 < x < 400) = \text{Pnorm}(z < .61) - \text{Pnorm}(z < -.92)$

$= .7291 - .1788 = \boxed{.5503}$

6. Assume $B_1, B_2, B_3, \ldots, B_n$ are pairwise mutually exclusive events which together cover the sample space. Show, for any event A, that

$$p(B_i|A) = \frac{p(B_i)p(A|B_i)}{p(B_1)p(A|B_1) + p(B_2)p(A|B_2) + \cdots + p(B_n)p(A|B_n)}$$

$$p(B_i|A) = \frac{p(B_i \cap A)}{p(A)}$$ by def'n of conditional probability

$$= \frac{p(B_i)\,p(A|B_i)}{p(A)}$$ by product rule.

$$A = (B_1 \cap A) \cup (B_2 \cap A) \cup (B_3 \cap A) \cup \cdots \cup (B_n \cap A)$$

$$p(A) = p(B_1 \cap A) + p(B_2 \cap A) + p(B_3 \cap A) + \cdots + p(B_n \cap A)$$

since $B_1, B_2, B_3, \ldots, B_n$ are pairwise mutually exclusive.

so $$\frac{p(B_i)p(A|B_i)}{p(A)} = \frac{p(B_i)p(A|B_i)}{p(B_1 \cap A) + p(B_2 \cap A) + \cdots + p(B_n \cap A)}$$

$$= \frac{p(B_i)\,p(A|B_i)}{p(B_1)p(A|B_1) + p(B_2)p(A|B_2) + \cdots + p(B_n)\,p(A|B_n)} \quad \square.$$