Exam 1 Review Sheet

Counting

Permutations: $P(n,k)$ =the number of length k sequences (without repetition) from a set of size n. $P(n,k) = n!/(n-k)!$

Combinations: $\binom{n}{k}$ =the number of size k subsets of a set of size n. $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

Multiplication principle: n things are to be decided, there are m_i choices for the i^{th} decision. There are $m_1 \cdot m_2 \cdot m_3 \cdots m_n$ ways to make the decisions.

Sets: $n(A)$ is the size of the set A. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

Probability

Probability distributions: $x_1, x_2, x_3, \ldots, x_n$ are all the possible outcomes. Then $0 \leq p(x_i) \leq 1$ for all x_i and $\sum p(x_i) = 1$.

If each outcome is equally likely, for any event A, $p(A) = n(A)/n(S)$ where S is the sample space.

$p(A) + p(A') = 1$

Conditional Probability: $p(A|B) = \frac{p(A\cap B)}{p(B)}$

Product Rule: $p(A \cap B) = p(A)p(B|A)$. For independent events, $p(A \cap B) = p(A)p(B)$.

Bayes' Theorem: Where $B_1, B_2, B_3, \ldots, B_k$ are mutually exclusive events so that they're union is the whole sample space,

$$p(B_i|A) = \frac{p(B_i)p(A|B_i)}{p(B_1)p(A|B_1) + p(B_2)p(A|B_2) + \cdots + p(B_k)p(A|B_k)}$$

Expected Value: $E[X] = \sum x_i p(x_i)$

Statistics

Sample mean: $\overline{x} = \frac{1}{n}\sum x_i$

Population mean: $E[x] = \mu = \sum xp(x)$

Median: Item $n/2$ when the outcomes are listed in increasing order.

Mode: The most frequent outcome

Sample Variance: $s^2 = \frac{1}{n-1}\sum(x-\overline{x})^2$

Sample Standard Deviation: $s = \sqrt{s^2}$

Population Variance: $\sigma^2 = E[(x-\overline{x})^2]$

Population Standard Deviation: $\sigma = \sqrt{\sigma^2}$

Normal Distribution: e^{-x^2} has $\mu = 0$ and $\sigma = 1$. $p_{norm}(x < a)$ is given in tables.

z-scores: $z = \frac{x-\mu}{\sigma}$. $p(x < a) = p_{norm}\left(z < \frac{a-\mu}{\sigma}\right)$ when x has a normal distribution with mean μ and standard deviation σ.

Binomial Probability

If the probability of success is p, then the probability of k successes in n trials is $\binom{n}{k}p^k(1-p)^{n-k}$.

The expected number of successes is np.

Binomial distributions are nicely approximated with normal distributions of mean np and standard deviation $\sqrt{(np(1-p)}$.

Data Organization

Trees organize probability into mutually exclusive events.

histograms display relative size of events.

Venn Diagrams divide the world into events.

Frequency Distributions make data easier to work with.