Practice Problems for Test 2

Note: The test will have at most ten problems. As usual calculators will not be permitted.

Problem 1. Give an example of a sentence that *is* a proposition and one that *is not* a proposition. Explain why. (Note: Proposition and statement are two words that mean the same thing.)

Problem 2. Let p denote the statement "Jack committed the crime" and let q denote the statement "Jack is friendly." Write the statement "Jack committed the crime and he is not friendly" in logical symbols.

Problem 3. Construct a truth table for the statement: $(\sim p \wedge q) \to p$.

Problem 4. Give an example (in English) of an implication. Also give its converse, contrapositive and negation. Which of these statements are logically equivalent? Why?

Problem 5. In TWO WAYS show that $[(p \wedge r) \to q] \Leftrightarrow [q \vee (\sim p \vee \sim r)]$.

Problem 6. Show that the following argument is valid.

If Bela is a professional wrestler, then he is large or strong. Bela is not large and he is not strong. Therefore, Bela is not a professional wrestler.

Problem 7. State whether or not the games having the following payoff matrices are strictly determined. If so, give the optimal pure strategies and the values of the games.

$$\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} \quad \begin{bmatrix} 4 & 1 & 0 \\ 5 & 4 & 3 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 5 & -3 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & -2 & -1 \\ 1 & 0 & 3 \\ -3 & -2 & -2 \end{bmatrix}$$

Problem 8. In the following game each player chooses an integer. If the sum is even, R pays C one dollar; if the sum is odd, C pays R one dollar. What is the payoff matrix for this game? Is this game strictly determined?

Problem 9. Suppose a game has payoff matrix $\begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & 0 \end{bmatrix}$ and R plays $\begin{bmatrix} .9 & .1 \end{bmatrix}$. Which of $\begin{bmatrix} .2 \\ .3 \\ .5 \end{bmatrix}$ and $\begin{bmatrix} .7 \\ .2 \\ .1 \end{bmatrix}$ is better for C?

Problem 10. A game is played by two people. R conceals either a one dollar bill or a two dollar bill in his hand. C guesses 1 or 2, and wins the bill if she guesses its number.

1. What is the payoff matrix for this game?
2. Determine C's optimal strategy.
3. Determine the value of the game.

Problem 11. Pick a voting method and describe how it works. Also describe one of its advantages or one of its disadvantages. Provide a simple example to illustrate this advantage/disadvantage if possible. You may also refer to one of the examples from class.

Problem 12. Problem 7abc(i) from the handout is a good practice problem. Also if the vote is tallied by approval, and each person votes for their favorite two restaurants, what is the outcome?