Your Name Here

Math 71
Homework 8

pXXX, #1 Let R be an integral domain and $I \subset R$ an ideal. Let $f \mapsto \overline{f}$ denote the reduction homomorphism $R[x] \to (R/I)[x]$. For $f \in R[x]$, show that $R[x]/(I, f) \cong (R/I)[x]/(\overline{f})$. Hint: You should verify that there is a natural surjective homomorphism $R[x] \to (R/I)[x] \to (R/I)[x]/(\overline{f})$. Give a careful argument to show that the map has the desired kernel.

A standard example is take $f \in \mathbb{Z}[x]$. Then $\mathbb{Z}[x]/(n, f(x)) \cong \mathbb{Z}/n\mathbb{Z}[x]/(\overline{f})$ which gives you information about the ideal (n, f) in $\mathbb{Z}[x]$ in terms of $\mathbb{Z}/n\mathbb{Z}[x]/(\overline{f})$. You will no doubt find other uses.

p298. #5. Prove that (x, y) and $(x, y, 2)$ are prime ideals in $\mathbb{Z}[x, y]$, but only the latter is maximal.

p298, #13. Show that the rings $F[x, y]/(y^2 - x)$ and $F[x, y]/(y^2 - x^2)$ are not isomorphic for any field F.

p301, #3 Let F be a field and x and indeterminate, $f \in F[x]$. Show that $F[x]/(f)$ is a field if and only if f is irreducible in $F[x]$. (Use Proposition 7 in §8.2)

p301, #5 If $p(x) \in F[x]$, exhibit all the ideals in $F[x]/(p(x))$ using the factorization of p in $F[x]$.

p301, #7 Determine all the ideals of the ring $\mathbb{Z}[x]/(2, x^3 + 1)$.